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Mathematical modeling of key fertility indicators based on Nepal census 2021 data

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Abstract

Inefficient registration of vital events and the geographical remoteness of many regions in developing countries often hinder the regular availability of reliable data. Vital event data—such as statistics on mortality, fertility, and migration—are often limited and inadequate across developing countries. In many of these nations, civil registration and vital statistics (CRVS) systems function inefficiently, and reliable data are very limited. Hence, mathematical modeling of such data is of great importance—not only because these statistics serve as key development indicators, but also because they are widely used by policymakers and planners. Fertility and mortality figures are among the most critical vital statistics. In many cases, fertility is of particular interest to demographers, sometimes even more than mortality, because death occurs only once in an individual's lifetime, whereas fertility events can occur multiple times. In this paper, the evolving fertility dynamics of Nepal is explained by developing parsimonious mathematical models. Firstly, the behavior of Parity progression ratio (PPR) and Total Fertility Rate are elucidated. This is based on the number of children ever born to a married woman data of Census 2021. It is found that the PPR in younger women for the age group 20 to 35 is lower than that of women in age group 35 to 50. Further, the latent trends and gender disparity in the behavior of Age at First Marriage are also explained. The developed models have explained the inherent behaviors and patterns of these fertility measures with a high degree of accuracy, across different regions of Nepal. The methods developed here can also be used to explain the fertility scenario in other countries of south Asia and Africa.

Keywords: Parity progression ratio; Birth order; Total fertility rate; Marital fertility; Age at first marriage.

1. Introduction

Fertility is a key demographic factor that shapes a nation's socioeconomic landscape. It is a multidimensional demographic phenomenon that can be examined not only through age-specific fertility rates, but also by considering indicators such as PPRs and Age at first marriage. Over the past several decades, fertility patterns have changed considerably, influenced by a wide range of social, economic, and health-related factors.

Nepal has experienced substantial demographic transformation over recent decades, most notably reflected in a marked decline in fertility rates. This shift has been largely driven by socio-economic developments, including the expansion of healthcare services, greater access to education for women, rapid urbanization, and changing social norms. In Nepal, a substantial share of the working-age population continues to migrate abroad in search of better employment opportunities, and this has contributed to a marked decline in fertility levels in recent years. This steadily declining fertility rate, according to Nepal Census 2021, has reached a below replacement level [1].

In general, more advanced developing countries tend to have smaller family sizes compared to the least developed ones. According to Nepal Census 2021, the Total fertility rate (TFR), which stood at around 6.39 in the 1980s, had fallen to 1.94 by 2021 [1]. Likewise, the Crude birth rate (CBR), which measures the number of live births per 1,000 people per year, has fallen sharply from about 45–50 in the 1950s to 14.2 in 2021. This decline reflects notable progress in healthcare, family planning, and overall economic development. Despite this improvement, rural areas still report a higher CBR (15.7) compared to urban areas (13.6). Similarly, between 2001 and 2021, age specific fertility rates (ASFR) declined overall, with the most significant reductions observed among women aged 25–39. Adolescent fertility (ages 15–19) also decreased markedly, falling from 71 births per 1,000 women in 2001 to 47 in 2021, highlighting the growing influence of education and awareness. Although women aged 20–24 continue to exhibit the highest fertility levels, rates in this group have also dropped from 203 per 1,000 in 2001 to 149 in 2021. Thus, age-specific fertility patterns indicate shifts across all reproductive age groups.

Also, the Singulate mean age at marriage (SMAM) has risen for both men and women over the past two decades. For males, it increased from 22.9 years in 2001 to 25.5 years in 2021, while for females it rose from 19.5 to 21.8 years over the same period. SMAM is consistently higher in urban areas, suggesting that economic security, educational attainment, and career aspirations contribute to postponing marriage and childbearing. Nepal is home to 1,688,886 single mothers, with the largest numbers recorded in Madhesh (370,611) and Lumbini (352,978). The number of widows is also substantially higher than that of widowers, reflecting women's longer life expectancy and differing remarriage patterns.

A diverse set of models have been developed to analyze and project population demographic behaviors, including fertility patterns. Some of the notable works are the following. An analysis of patterns in the fertility data, by developing PPR models and estimating the rural urban fertility differentials, was conducted by Devkota [2, 3]. The Age specific fertility rate (ASFR) of Nepal was also explained through various deterministic and probabilistic models, developed by Devkota [4, 5]. Similarly, Shrivastav et al. compared several models (P-K model, Gompertz, skew-normal, etc.) for India's ASFR and proposed flexible alternatives to capture unimodal fertility curves [6]. Likewise, Ibeji et al. applied count regression models (Poisson, negative binomial, generalized Poisson) to Nige-

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rian Demographic and Health Survey (DHS) data [7]. Also, Hussen used multilevel negative binomial regression to account for hierarchical structure in fertility determinants data [8]. Tomal et al. applied a Bayesian Poisson regression model to count data (children ever born) to identify socio-economic predictors in Bangladesh [9]. Gaire et al. surveyed mathematical models of age-specific fertility rates and explored alternative distributions suited for unimodal fertility patterns in Nepal and Malaysia, as well [10].

2. Materials and methods

Parity progression

Fertility can be described not only by a woman's progression through successive ages, but also by her transition from one parity level to the next. The latter perspective is captured by PPRs. A woman's parity refers to the total number of live births she has had. Thus, the PPR from parity i to $i+1$ is defined as the proportion of women in a cohort who, having already had i live births, subsequently go on to have at least one additional birth [11].

A cohort's TFR can be estimated using census or survey data on parity, meaning the number of live births a woman has had. A PPR is the proportion of women who, having already borne a certain number of children, go on to have at least one more child.

PPR for a cohort that has completed childbearing is calculated in the following manner. Let W_i be the number of women at parity i, and let

$$P_i = \sum_{a=i}^{I} W_a$$

be the number of women at parity i or greater, and $B_i = P_i$ for all $i > 0$ the number of births of parity i. Then:

$$\mathrm{PPR}(i,\, i+1) = \frac{P_{i+1}}{P_i} = \frac{B_{i+1}}{B_i} \tag{1}$$

$$\mathrm{PPR}(0,\, i) = \frac{P_i}{P_0} = \prod_{a=0}^{i-1} \mathrm{PPR}(a,\, a+1) \tag{2}$$

$$\mathrm{TFR} = \frac{B}{W} = \sum_{i=1}^{I} \frac{B_i}{W} = \sum_{i=1}^{I} \mathrm{PPR}(0,\, i) \tag{3}$$

Thus, for women who have completed their childbearing years, the average number of children ever born in the cohort corresponds to the cohort's TFR, provided the data are accurately reported and there are no differences in mortality or migration across parity levels.

Elasticity of fertility measures

Elasticity quantifies how responsive one variable is to changes in another. It is expressed as the percentage change in the former associated with a one-percent change in the latter [11].

Mathematically, for a function $y = f(x)$, the elasticity is defined as:

$$E_x = \frac{dy}{dx} \cdot \frac{x}{y}$$

Elasticity is dimensionless and quantifies how responsive y is to changes in x.

The elasticity of $\mathrm{PPR}(0, i)$ for changes in i over women of different age groups is compared. Similarly, the elasticity of the probability function of Age at first marriage is computed for the measurement of gender differential in age at marriage. Thus, elasticity is estimated to examine gender-specific and age-specific differentials in the interpretation of fertility dynamics.

3. Results and discussion

The results obtained here are primarily based on Nepal Census 2021 data, namely ever married female population 15–49 years of age by number of children ever born alive and age of mothers, and married population aged 10 years and above by Age at first marriage [12].

Based on ever married female population 15–49 years of age by number of children ever born alive data and using Equations (1)–(3), Table 1 was obtained. This table computes the PPR and then TFR for the whole of Nepal. As seen from Table 1, the TFR value obtained from PPR is 2.082, whereas the census TFR is 1.94. The difference in these TFR values indicates a differential in mortality and migration by parity and perhaps inaccurate reporting.

Table 1: Computation of PPR and TFR of whole Nepal, based on Nepal census 2021

i	W_i	P_i	B_i	$\mathrm{PPR}(i, i{+}1)$	$\mathrm{PPR}(0, i)$
0	736 499	6 145 039		0.8801	0.8801
1	1 419 503	5 408 540	5 408 540	0.7375	0.6491
2	2 027 343	3 989 037	3 989 037	0.4918	0.3192
3	1 100 021	1 961 694	1 961 694	0.4392	0.1402
4	508 747	861 673	861 673	0.4096	0.0574
5	211 983	352 926	352 926	0.3994	0.0229
6	86 725	140 943	140 943	0.3847	0.0088
7	33 660	54 218	54 218	0.3792	0.0033
8	14 378	20 558	20 558	0.3006	0.0010
9	6 180	6 180	6 180	0	0
	Total $W = 6\,145\,039$		12 795 769	TFR	2.082
		Census TFR			1.94

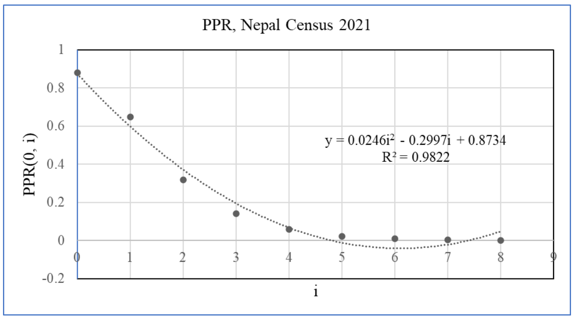

Figure 1: Observed and predicted PPR for whole Nepal, Nepal Census 2021.

The progression of $\mathrm{PPR}(0, i)$ over different values of parity i, given in Table 1, is explained by a polynomial equation in Figure 1. As seen from Figure 1, this polynomial model explains the movement in $\mathrm{PPR}(0, i)$ over different values of i with an accuracy of 98.22 percent. Here y represents $\mathrm{PPR}(0, i)$ and is the dependent variable; the independent variable in the equation is parity i.

The progression of PPR in the women in age groups 20–35 years and 35–49 years is given in Figure 2. Similar to Table 1, these PPR values were computed for women of age groups 20–35 and 35–49 separately, using Equations (1)–(3). As seen from this figure, there is a sharp decline in PPR in the age group 20–35 as compared to 35–49, indicating declining fertility. The TFR values obtained for the age groups 20–35 and 35–49 are 1.650 and 2.716, respectively.

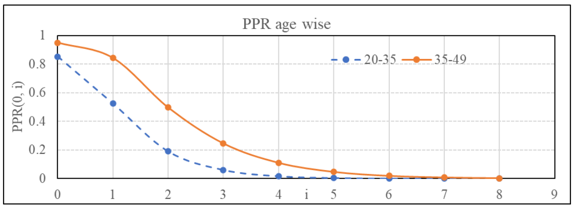

Figure 2: Observed and predicted PPR for age groups 20–35 and 35–49, Nepal Census 2021.

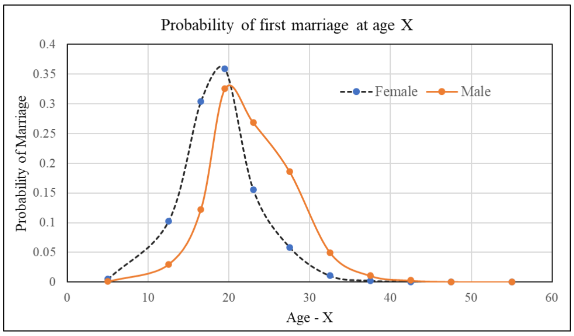

Figure 3: Probability of marriage at age X for whole Nepal, Nepal Census 2021.

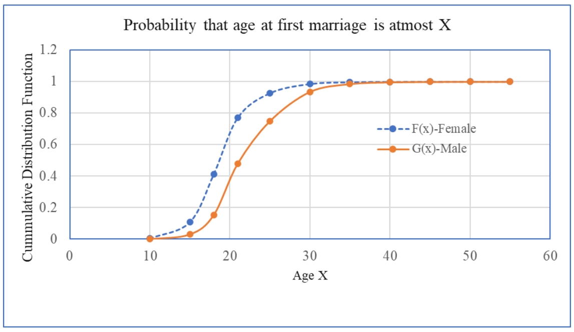

Figure 4: CDF of Age at first marriage for whole Nepal, Nepal Census 2021.

Similarly, Table 2 is obtained from the Census 2021 data on the age of mothers and married population aged 10 years and above by Age at first marriage. This table also gives the probability of marriage according to the different age groups for male and female, for the whole of Nepal. Here, $p_1(x)$ and $p_2(x)$ are the probability distribution of first marriage at age X, and $G(x)$ and $F(x)$ are the cumulative distribution function (CDF) of this probability distribution, for male and female respectively.

The age-specific probability of first marriage for both sexes at the national level in Nepal is presented in Figure 3. The figure shows that women have a substantially higher probability of entering first marriage at younger ages compared to men. Consistent with this pattern, the estimated expected Age at first marriage, $E(X)$, is 18.98 years for women, which is considerably lower than that for men (22.24 years). This pronounced gender gap reflects prevailing social norms, educational trajectories, and economic roles that continue to encourage earlier marriage among women.

Furthermore, Figure 4 illustrates gender differentials in the cumulative distribution function (CDF) across different parameter values, highlighting systematic differences in the timing and pace of marital transitions between men and women. Together, these figures underscore the persistence of gendered patterns in marriage behavior in Nepal, with important implications for fertility, education, and women's empowerment.

The $\mathrm{PPR}(0,i)$ shown in Figure 2 for women of age groups 20–35 and 35–49 is explained with the help of the following polynomial models with $R^2 = 0.956$ and $R^2 = 0.9794$, respectively:

$$\mathrm{PPR}(0,i) = y = 0.0265i^2 - 0.302i + 0.7917 \tag{4}$$

$$\mathrm{PPR}(0,i) = y = 0.0215i^2 - 0.266i + 0.8627 \tag{5}$$

Similarly, the CDF displayed in Figure 4 for male and female gender is explained by Equations (6) and (7) with $R^2 = 0.9815$ and $R^2 = 0.955$, respectively:

$$G(x) = y = 0.000003x^4 - 0.0004x^3 + 0.0172x^2 - 0.243x + 1.0452 \tag{6}$$

$$F(x) = y = 0.000002x^4 - 0.0002x^3 + 0.0068x^2 - 0.0234x - 0.3167 \tag{7}$$

The estimated elasticities capture gender and age differentials, providing insights into the mechanisms driving fertility dynamics. Elasticity estimates derived from the Nepal Census 2021 data offer valuable insights into how fertility behavior responds to changes in reproductive timing amid rising ages at marriage and overall fertility decline.

The absolute values of elasticity of Equations (4) and (5) are given in Table 3. The estimated elasticity of the PPR, $\mathrm{PPR}(0,i)$, is higher among women aged 20–35 than among those aged 35–49. Based on Nepal Census 2021 data, this indicates that parity progression during early and mid-reproductive ages is more responsive to socioeconomic, demographic, and contextual factors than at later ages. Women in the 20–35 age group are more likely to adjust their childbearing behavior in response to changes in education, employment opportunities, access to family planning, and household economic conditions. In contrast, lower elasticity among women aged 35–49 suggests that fertility behavior in later reproductive years is relatively more stable and less sensitive to changing circumstances, reflecting completed or near-completed family size, biological constraints, and entrenched reproductive preferences. Overall, this pattern highlights the importance of early and mid-life interventions in shaping fertility trajectories in Nepal and underscores the critical role of younger cohorts in driving recent fertility transitions.

Table 3: Absolute values of elasticity of $\mathrm{PPR}(0,i)$ w.r.t. i

i	Age Group of Women	
	20–35	35–49
0	0	0
1	0.476	0.296
2	2.067	0.792
3	7.087	1.758
4	20.970	3.311
5	38.706	4.068
6	75.458	5.034
7	1609.068	66.547
8	29172.936	435.391

Table 2: Probability of marriage at age X for whole Nepal, based on Nepal Census 2021

Age (in Years)	X	Male	Female	Prob. Male $p_1(x)$	Prob. Female $p_2(x)$	$G(x)$	$F(x)$
Below 10 years	10	8 575	45 773	0.0010	0.0050	0.0012	0.0051
10–15	15	214 266	910 939	0.0301	0.1020	0.0313	0.1070
15–18	18	872 401	2 703 376	0.1230	0.3040	0.1540	0.4110
18–21	21	2 318 006	3 193 620	0.3260	0.3590	0.4800	0.7700
21–25	25	1 908 640	1 384 650	0.2680	0.1550	0.7480	0.9250
25–30	30	1 322 893	522 708	0.1860	0.0590	0.9340	0.9840
30–35	35	355 450	95 066	0.0500	0.0110	0.9840	0.9940
35–40	40	76 590	22 477	0.0110	0.0030	0.9940	0.9970
40–45	45	20 210	6 614	0.0030	0.0007	0.9970	0.9980
45–50	50	4 915	1 502	0.0007	0.0002	0.9970	0.9980
50 and above	55	2 813	833	0.0004	9.4×10^{-5}	0.9980	0.9980
Not stated		11 900	18 653	0.0010	0.0050	—	—
Total		7 116 659	8 906 211				

Table 4: Absolute values of Elasticity of CDF w.r.t. Age

Age X	Female	Male
10	117.912	34.634
15	10.138	0.984
18	3.226	0.245
21	1.956	1.056
25	1.800	3.088
30	1.848	7.887
35	2.133	16.105
40	2.913	28.496
45	4.628	45.812
50	7.847	68.801

The estimated elasticity of the CDF is higher for females at younger ages, while for males it is higher at older ages, as indicated in Table 4. Using Nepal Census 2021 data, this pattern suggests that women's probability of marrying by a given age is more sensitive to socioeconomic and demographic factors—such as educational attainment, household characteristics, and cultural norms—during adolescence and early adulthood, particularly up to age 21. This reflects the continued prevalence of early marriage among women in several regions of Nepal, despite recent policy efforts and social change. In contrast, from approximately age 25 onwards, the probability of marriage becomes more elastic for men than for women. This implies that male marriage timing in Nepal is more strongly influenced at later ages by factors such as labor market participation, income stability, migration for employment, and housing conditions. Delayed and more variable marriage among men is consistent with rising educational participation and overseas labor migration observed in the 2021 Census.

Overall, these results highlight a gender-differentiated pattern in marriage timing in Nepal, where women's marital transitions are more responsive to structural and normative constraints at younger ages, while men's transitions are increasingly shaped by economic and mobility-related factors in adulthood. This differential sensitivity has important implications for fertility behavior and family formation patterns in the country.

A discrepancy is observed between the TFR values derived from PPR and those obtained from Census data across different ecological zones of Nepal, in Table 5. The observed differences in these TFR values suggest disparities in mortality and migration across parity levels, as well as the possibility of reporting inaccuracies. A similar discrepancy is also observed in Table 6, highlighting these values in the seven provinces of Nepal.

In Tables 5 and 6, the third column reports the TFR estimated using the PPR method, while the fourth column presents the official TFR from the National Population and Housing Census 2021 conducted by the National Statistics Office (Nepal). Columns five and six give the quadratic polynomial regression equations and their corresponding R^2 values, which describe the progression of PPRs across successive parities i and indicate the goodness of fit of the models.

Overall, TFR estimates obtained from PPR estimates are slightly higher than the census values in most regions of Nepal, with the notable exceptions of the Tarai and Madhesh. The TFR obtained from PPR is 2.268 in rural areas and 1.992 in urban areas, confirming higher fertility in rural settings.

Across ecological zones as seen from Table 5, the Mountain region records a PPR-based TFR of 2.375 compared with the census value of 2.24. It can also be seen that in the Hill region, it is 1.963 and 1.64, and in the Tarai region, it is 2.139 and 2.20, respectively.

It can be seen from Table 6 that the largest discrepancy is observed in Bagmati province, where the PPR-based TFR is 1.791 compared with the census estimate of 1.37. Further, Bagmati province also shows the lowest census TFR and correspondingly low PPR-based TFR in whole Nepal. In the Tarai, the two estimates seem to be closely aligned (2.139 and 2.20, respectively).

At the provincial level, Madhesh Province reports the highest census TFR (2.85), and highest PPR-based TFR (2.419). Karnali Province and Sudurpashchim Province also exhibit relatively high fertility levels.

It can also be seen from Tables 5 and 6 that in Tarai and Madhesh, the census TFR values are higher than those of the PPR-based TFR. This pattern is unlike other regions shown in the same tables. This can be attributed to cross-border migration, as Nepal shares an open border with India.

The R^2 values exceed 0.95 for most provinces and reach as high as 0.992 in Karnali, indicating an excellent fit of the quadratic models to the observed PPR data. This suggests that the polynomial regression equations adequately capture parity progression patterns across regions.

4. Conclusion

This study has developed parsimonious mathematical models to explain and quantify key fertility indicators in Nepal using Census 2021 data. The polynomial models for $\mathrm{PPR}(0, i)$ achieve high goodness of fit ($R^2 > 0.95$) across all ecological zones and provinces, demonstrating that quadratic regression adequately captures parity progression patterns. The TFR derived from PPR (2.082 nationally) is slightly higher than the Census value (1.94),

Table 5: PPR in different ecological zones of Nepal, Census 2021

Sr. No.	Region	$\mathrm{PPR}(0,i)$ TFR	Census TFR	$y = \mathrm{PPR}(0,i)$	R^2
1	Mountain	2.375	2.24	$y = 0.0215i^2 - 0.280i + 0.899$	0.9924
2	Hill	1.963	1.64	$y = 0.030i^2 - 0.330i + 0.872$	0.9845
3	Tarai	2.139	2.20	$y = 0.024i^2 - 0.300i + 0.887$	0.984

Table 6: PPR in the seven provinces of Nepal, Census 2021

Sr. No.	Region	$\mathrm{PPR}(0,i)$ TFR	Census TFR	$y = \mathrm{PPR}(0,i)$	R^2
1	Koshi	1.926	1.81	$y = 0.026i^2 - 0.309i + 0.855$	0.971
2	Madhesh	2.419	2.85	$y = 0.022i^2 - 0.289i + 0.926$	0.987
3	Bagmati	1.791	1.37	$y = 0.027i^2 - 0.309i + 0.828$	0.9588
4	Gandaki	1.859	1.54	$y = 0.027i^2 - 0.314i + 0.851$	0.964
5	Lumbini	2.070	1.84	$y = 0.025i^2 - 0.299i + 0.869$	0.981
6	Karnali	2.412	2.19	$y = 0.021i^2 - 0.279i + 0.904$	0.992
7	Sudurpashchim	2.294	1.88	$y = 0.023i^2 - 0.289i + 0.899$	0.990

reflecting parity-specific differences in mortality, migration, and possible reporting inaccuracies.

Age-group comparisons reveal a sharper decline in PPR among women aged 20–35 (TFR = 1.650) relative to those aged 35–49 (TFR = 2.716), and the higher elasticity of PPR in the younger cohort confirms that early-to-mid reproductive-age fertility is more sensitive to socioeconomic and structural changes. Gender differentials in marriage timing are clearly captured by the CDF models: female marriage probability is more elastic at younger ages, while male marriage probability becomes more elastic beyond age 25, consistent with male labor migration and delayed economic stability.

At the sub-national level, Madhesh and Karnali show the highest fertility, while Bagmati shows the largest gap between PPR-based and Census TFR estimates. The reversed pattern in Tarai and Madhesh—where Census TFR exceeds the PPR estimate—is attributed to cross-border mobility with India. The methods presented here are transferable to other South Asian and African countries where CRVS data are limited, offering a robust analytical framework for monitoring demographic transitions from census-based parity data.

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