



SH-wave propagation in two micro-morphic contact half-spaces.

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Abstract

In this work, the propagation of SH (Shear Horizontal) waves over two interconnected micro-morphic half spaces is investigated. By transforming the fundamental relations into the xy-plane and applying the well-known "plane harmonic solution" approach to solve them, the secular equation was produced. Three more waves are shown that are not present in classical elasticity. The study investigates how micro-morphic properties affect wave propagation, providing insights into how SH waves behave in non-classical materials.

Keywords: Micro-morphic material; SH waves; Contact half-spaces; Secular relation.

1. Introduction

The distribution of couplings throughout the surface where various components of a continuum interact mechanically is ignored by the classical theory of elasticity. The premise of the micro-polar hypothesis, which was proposed by Eringen [1, 2] and Eringen [3], is that the medium's micro motion is limited to micro-rotation alone. Micro-motion is made up of micro-translation and micro-rotation, according to Eringen's [4] theory of micro-morphic materials. Twelve second-order partial differential equations with twelve unknowns and eighteen elastic constants are needed to analyze the mechanical behavior of a micro-morphic material. By applying to the micro-elastic solid the idea of coincidence of primary directions of stress and strain in classical elasticity, Koh [5] created a more straightforward theory. By applying a certain type of micro-isotropy, Koh [5] was able to derive unique constraints on the elastic moduli, which in the exceptional situation reduced number 18 to 10. One simply needs to take into account nine equations in nine unknowns in a later formulation of the problem with regard to the major direction of the micro-strain.

Koh's [5] theory is referred to as micro-isotropic, micro-elastic theory. The equations of micro-polar elasticity are obtained by assuming that the micro-motion is limited to micro-rotation and that the stress moment tensor exhibits a specific type of anti-symmetry. Some have examined wave propagation when the distance between the point source and the plane is finite. Sommerfeld [6], Jeffery [7], Muskantand [8], and others have examined the propagation of plane waves in two semi-infinite mediums separated by a plane interface. In their study, M. Parameshwar Rao and B. Kesava Rao [9] examine wave propagation in a semi-infinite micropolar isotropic elastic solid superimposed on another micropolar elastic solid. Researchers S.K. Tomar and S.L. Saini [10] have examined the reflection and refraction of SH waves across a corrugated interface between two-dimensional transversely isotropic half spaces. Somaiah and Ravikumar [11] studied the wave propagation at half-space. The impacts of angular rotation, initial compression under impedance boundary conditions on Love-type SH-wave propagation was studied by Somaiah [12]. In recent, rotation,

initial compression, surface stress and sliding contact effects are discussed by Somaiah et al [13, 14].

In this paper we discuss the SH waves in two micro-morphic elastic half spaces. This problem is of geophysical interest, particularly in investigations concerned with earthquakes and other phenomena in seismology. The initial approximation to the real situation can be made by considering the earth as being composed of multiple layers, each of which has constant physical qualities, while the propagation characteristics of the earth change with depth. We noticed that there are three more waves when taking time harmonic waves into account. These three waves are dispersive and depend only on micro-morphic constants A_5 and B_2 and that are other than λ and μ .

2. Basic equations:

The basic equations for micro-isotropic, micro elastic solids are obtained by Koh [5], Parameswaran and Koh [15] are given as follows. The constitutive equations are

$$t_{(km)} = A_1 e_{pp} \delta_{km} + 2A_2 e_{km}, \quad (1)$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \epsilon_{pkm} (r_p + \phi_p), \quad (2)$$

$$\sigma_{(km)} = -A_4 \phi_{pp} \delta_{km} - 2A_5 \phi_{(km)}, \quad (3)$$

$$t_{k(mn)} = B_1 \phi_{(pp,k)} \delta_{mn} + 2B_2 \phi_{(mn,k)}, \quad (4)$$

$$m_{(kl)} = -2 (B_3 \phi_{l,k} + B_4 \phi_{(k,l)} + B_5 \phi_{p,p} \delta_{kl}) \quad (5)$$

where $()$ denotes symmetric part and $[]$ denotes anti-symmetric part and

$$\begin{aligned} A_1 &= \lambda + \sigma_1, & B_1 &= \tau_3, \\ A_2 &= \mu + \sigma_2, & 2B_2 &= \tau_7 + \tau_{10} = \tau_2, \\ A_3 &= \sigma_5, & B_3 &= 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10}, \\ A_4 &= -\sigma_1, & B_4 &= -2\tau_4, \\ A_5 &= -\sigma_2, & B_5 &= -2\tau_9 \end{aligned} \quad (6)$$

Subject to conditions

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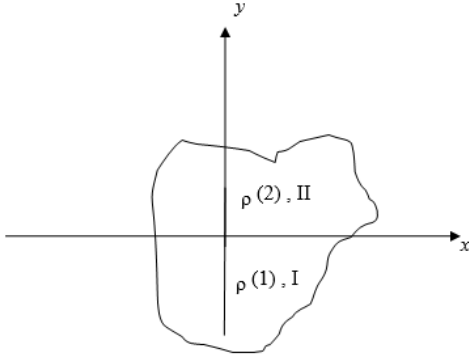


Figure 1: Geometry of the problem.

$$\begin{aligned} 3A_1 + 2A_2 &> 0, & A_2 &> 0, & A_3 &> 0, \\ 3A_4 + 2A_5 &> 0, & A_5 &> 0, & B_3 &> 0, \\ -B_3 &< B_4 < B_5, & B_3 + B_4 + B_5 &> 0 \end{aligned} \quad (7)$$

where $\phi_p = \frac{1}{2} \epsilon_{pkm} \phi_{km}$, $r_k = \frac{1}{2} \epsilon_{pkm} u_{m,k}$ and the couple stress tensor m_{kp} and the body couples l_p are respectively given by

$m_{kp} = \epsilon_{pkm} t_{kmn}$, $l_p = \epsilon_{pmn} f_{mn}$, and λ, μ are Lamé's (classical material) constants, $\sigma_1, \sigma_2, \sigma_5$ are micro-isotropic constants, $\tau_2, \tau_3, \tau_4, \tau_7, \tau_9$ and τ_{10} . Further ϵ_{pnm} represents the permutation symbol.

$$\begin{aligned} (A_1 + A_2 - A_3) u_{p,pm} + (A_2 + A_3) u_{m,pp} \\ + 2A_3 \epsilon_{pkm} \phi_{p,k} + \rho f_m = \rho \frac{\partial^2 u_m}{\partial t^2}, \end{aligned} \quad (8)$$

$$\begin{aligned} B_1 \phi_{pp,kk} \delta_{ij} + 2B_2 \phi_{(ij),kk} \\ - A_4 \phi_{pp} \delta_{ij} - 2A_5 \phi_{(ij)} \\ + \rho f_{(ij)} = \frac{1}{2} \rho j \frac{\partial^2 \phi_{(ij)}}{\partial t^2}, \end{aligned} \quad (9)$$

$$\begin{aligned} 2B_3 \phi_{p,mm} + 2(B_4 + B_5) \phi_{m,mp} \\ - 4A_3 (r_p + \phi_p) - \rho l_p = \rho j \frac{\partial^2 \phi_p}{\partial t^2}, \end{aligned} \quad (10)$$

$$(A_2^{(i)} + A_3^{(i)}) \left(\frac{\partial^2 w^{(i)}}{\partial x^2} + \frac{\partial^2 w^{(i)}}{\partial y^2} \right) + 2A_3^{(i)} \left(\frac{\partial \phi_1^{(i)}}{\partial y} - \frac{\partial \phi_2^{(i)}}{\partial x} \right) = \rho^{(i)} \frac{\partial^2 w^{(i)}}{\partial t^2}, \quad (13)$$

$$\left(\frac{\partial^2 \phi_1^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_2^{(i)}}{\partial x \partial y} \right) - 4A_3^{(i)} \left[\frac{1}{2} \frac{\partial w^{(i)}}{\partial y} + \phi_1^{(i)} \right] = \rho^{(i)} j^{(i)} \frac{\partial^2 \phi_1^{(i)}}{\partial t^2}, \quad (14)$$

$$\left(\frac{\partial^2 \phi_1^{(i)}}{\partial x \partial y} + \frac{\partial^2 \phi_2^{(i)}}{\partial y^2} \right) - 4A_3^{(i)} \left[-\frac{1}{2} \frac{\partial w^{(i)}}{\partial x} + \phi_2^{(i)} \right] = \rho^{(i)} j^{(0)} \frac{\partial^2 \phi_2^{(i)}}{\partial t^2}. \quad (15)$$

$$B_1^{(i)} \left(\frac{\partial^2 \phi_{33}^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_{33}^{(i)}}{\partial y^2} \right) - A_4^{(i)} \phi_{33}^{(i)} = 0, \quad (16)$$

$$B_1^{(i)} \left[\frac{\partial^2 \phi_{33}^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_{33}^{(i)}}{\partial y^2} \right] + 2B_2^{(i)} \left[\frac{\partial^2 \phi_{33}^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_{33}^{(i)}}{\partial y^2} \right] - A_4 \phi_{33}^{(i)} - 2A_5 \phi_{33}^{(i)} = \frac{1}{2} \rho^{(i)} j^{(i)} \frac{\partial^2 \phi_{33}^{(i)}}{\partial t^2}, \quad (17)$$

$$2B_2^{(i)} \left[\frac{\partial^2 \phi_{(32)}^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_{(32)}^{(i)}}{\partial y^2} \right] - 2A_5 \phi_{32}^{(i)} = \frac{1}{2} \rho^{(i)} j^{(i)} \frac{\partial^2 \phi_{32}^{(i)}}{\partial t^2}, \quad (18)$$

where suffix comma denotes partial derivatives with respect to space variable (x_k) and suffix repeated indices represents summation.

3. Formulation and solution of the problem:

Figure 1 illustrates that two micro-morphic half spaces with different mechanical properties perfectly welded along the x-axis.

For time harmonic propagation in the xy -plane, the components of macro displacements u, v, w , micro rotations ϕ_a ($1 \leq a \leq 3$) and macro-strains $\phi_{(ab)}$ ($1 \leq a, b \leq 3$) are considered as functions of x, y, t for both media and they are given by

$$\begin{aligned} u^{(1)} &= 0, & v^{(1)} &= 0, & w^{(1)} &= w^{(1)}(x, y, t), \quad (11) \\ \phi_1^{(1)} &= \phi_1^{(1)}(x, y, t), \\ \phi_2^{(1)} &= \phi_2^{(1)}(x, y, t), & \phi_3^{(1)} &= 0, \\ \phi_{(31)}^{(1)} &= \phi_{(31)}^{(1)}(x, y, t), \\ \phi_{33}^{(1)} &= \phi_{33}^{(1)}(x, y, t), \\ \phi_{11}^{(1)} &= 0, & \phi_{22}^{(1)} &= 0, & \phi_{(12)}^{(1)} &= 0 \end{aligned}$$

for $y < 0$ and

$$\begin{aligned} u^{(2)} &= 0, & v^{(2)} &= 0, \\ w^{(2)} &= w^{(2)}(x, y, t), \\ \phi_1^{(2)} &= \phi_1^{(2)}(x, y, t), & \phi_2^{(2)} &= \phi_2^{(2)}(x, y, t), & \phi_3^{(2)} &= 0, \\ \phi_{31}^{(2)} &= \phi_{31}^{(2)}(x, y, t), \\ \phi_{32}^{(2)} &= \phi_{32}^{(2)}(x, y, t), \\ \phi_{33}^{(2)} &= \phi_{33}^{(2)}(x, y, t), \\ \phi_{11}^{(2)} &= 0, & \phi_{22}^{(2)} &= 0, & \phi_{(12)}^{(2)} &= 0, \end{aligned} \quad (12)$$

for $y > 0$. Quantities with super suffix 1 denote corresponding to the medium I and that of super suffix 2 denote corresponding to the medium II ($y > 0$).

Under the absence of body forces and body couples the field equations (8) to (10) are reduce to

$$2B_2^{(i)} \left[\frac{\partial^2 \phi_{(31)}^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_{(31)}^{(i)}}{\partial y^2} \right] - 2A_5^{(i)} \phi_{(31)}^{(i)} = \frac{1}{2} \rho^{(i)} j^{(i)} \frac{\partial^2 \phi_{31}^{(i)}}{\partial t^2}. \quad (19)$$

In view of equation (16), the equation (17) reduces to

$$2B_2 \left(\frac{\partial^2 \phi_{33}^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_{33}^{(i)}}{\partial y^2} \right) - 2A_5 \phi_{33}^{(i)} = \frac{1}{2} \rho^{(i)} j^{(i)} \frac{\partial^2 \phi_{33}^{(i)}}{\partial t^2}. \quad (20)$$

where $i = 1, 2$. The field equations for medium I correspond to $i = 1$ and for medium II to $i = 2$.

The equations (13) to (15) are coupled in terms of $w^{(i)}$, $\phi_1^{(i)}$ and $\phi_2^{(i)}$. Assume the plane harmonic solution,

$$\begin{aligned} w^{(i)} &= A^{(i)} \exp(m^{(i)} y) \exp[iq(y - ct)], \\ \phi_1^{(i)} &= B^{(i)} \exp(m^{(i)} y) \exp[iq(y - ct)], \\ \phi_2^{(i)} &= C^{(i)} \exp(m^{(i)} y) \exp[iq(y - ct)], \end{aligned} \quad (21)$$

where $A^{(i)}$, $B^{(i)}$, $C^{(i)}$ are constants, q is the wave number and c is the phase velocity.

Substituting equations (21) into equations (13) to (15) we get

$$\left[(A_2^{(i)} + A_3^{(i)}) (m^{(i)2} - q^2) + \rho^{(i)} c^2 q^2 \right] A^{(i)} + 2A_3^{(i)} m^{(i)} B^{(i)} - 2A_3^{(i)} iqc = 0, \quad (22)$$

$$\begin{aligned} -2A_3^{(i)} m^{(i)} A^{(i)} + \left[2B_3^{(i)} (m^{(i)2} - q^2) - 2(B_4^{(i)} + B_5^{(i)}) q^2 - 4A_3^{(i)} + \rho^{(i)} j^{(i)} q^2 c^2 \right] B^{(i)} \\ + \left[2(B_4^{(i)} + B_5^{(i)}) m^{(i)} iq \right] C^{(i)} = 0, \end{aligned} \quad (23)$$

$$\begin{aligned} (2A_3^{(i)} iq) A^{(i)} + \left[2(B_4^{(i)} + B_5^{(i)}) iqm^{(i)} \right] B^{(i)} \\ + \left[2B_3^{(i)} (m^{(i)2} - q^2) + 2(B_4^{(i)} + B_5^{(i)}) m^{(i)2} - 4A_3^{(i)} + \rho^{(i)} j^{(i)} q^2 \right] C^{(i)} = 0. \end{aligned} \quad (24)$$

For $i = 1$ we have a system of homogeneous equations in $A^{(1)}$, $B^{(1)}$ and $C^{(1)}$.

Similarly for $i = 2$ we have another system in $A^{(2)}$, $B^{(2)}$ and $C^{(2)}$. For the existence of a non-trivial solution the determinant of the coefficient matrix must be zero:

$$|a_{ij}| = 0, \quad (25)$$

where

$$\begin{aligned} a_{11} &= (A_2^{(i)} + A_3^{(i)}) (m^{(i)2} - q^2) + \rho^{(i)} c^2 q^2, \\ a_{12} &= 2A_3^{(i)} m^{(i)}, \quad a_{13} = -2A_3^{(i)} iq, \\ a_{21} &= -2A_3^{(i)} m^{(i)}, \end{aligned}$$

$$\begin{aligned} a_{22} &= 2B_3^{(i)} (m^{(i)2} - q^2) - 2(B_4^{(i)} + B_5^{(i)}) q^2 \\ &\quad - 4A_3^{(i)} + \rho^{(i)} j^{(i)} c^2 q^2, \\ a_{23} &= 2(B_4^{(i)} + B_5^{(i)}) m^{(i)} iq, \quad a_{31} = 2A_3^{(i)} iq, \\ a_{32} &= 2(B_4^{(i)} + B_5^{(i)}) iqm^{(i)}, \\ a_{33} &= 2B_3^{(i)} (m^{(i)2} - q^2) + 2(B_4^{(i)} + B_5^{(i)}) m^{(i)2} \\ &\quad - 4A_3^{(i)} + \rho^{(i)} j^{(i)} q^2 c^2. \end{aligned} \quad (26)$$

Expanding the determinant we get

$$\begin{aligned} \left[j^{(1)} (\theta^{(1)} + \delta^{(1)}) (m^{(1)2} - q^2) + j^{(1)} q^{(2)} \xi^{(1)} - 2\epsilon^{(1)} \right] \\ \cdot \left[\theta^{(1)} j^{(1)} (m^{(1)2} - q^2) - 2\epsilon^{(1)} + \rho^{(1)} j^{(1)} \xi^{(1)} q^2 \right] \\ \cdot \left[\frac{C_1^{(1)2} - C_1^{(1)2}}{C_6^{(1)2}} m^{(1)2} - \frac{C_1^{(1)2} - C_2^{(1)2} - c^2}{C_6^{(1)2}} q^2 \right] \\ + \epsilon^{(1)2} (m^{(1)2} - q^2) = 0, \end{aligned} \quad (27)$$

where

$$\begin{aligned} C_1^{(1)2} &= \frac{A_1^{(1)} + 2A_2^{(1)}}{\rho^{(1)}}, \quad C_2^{(1)2} = \frac{A_1^{(1)} + A_2^{(1)} - A_3^{(1)}}{\rho^{(1)}}, \\ C_3^{(1)2} &= \frac{2A_3^{(1)}}{\rho^{(1)}}, \quad C_4^{(1)2} = \frac{2B_3^{(1)}}{\rho^{(1)} j^{(1)}}, \quad C_5^{(1)2} = \frac{2(B_4^{(1)} + B_5^{(1)})}{\rho^{(1)} j^{(1)}}, \\ C_6^{(1)2} &= \frac{A_2^{(1)}}{\rho^{(1)}}, \quad \xi^{(1)} = \frac{C^2}{C_6^{(1)2}}, \\ \epsilon^{(1)} &= \frac{C_3^{(1)2}}{C_6^{(1)2}}, \quad \theta^{(1)} = \frac{C_4^{(1)2}}{C_6^{(1)2}}, \quad \delta^{(1)} = \frac{C_5^{(1)2}}{C_6^{(1)2}}. \end{aligned} \quad (28)$$

Neglecting the $\epsilon^{(1)2}$ term in (27) we obtain approximate roots for $m^{(1)2}$ and we suppose these roots be b_1^2 , b_2^2 and b_3^2 . Thus,

$$b_1^2 = \frac{2\epsilon^{(1)}}{j^{(1)} + (\theta^{(1)} + \delta^{(1)})} + \left(1 - \frac{\xi^{(1)}}{\theta^{(1)} + \delta^{(1)}} \right) \theta^2, \quad (29)$$

$$b_2^2 = \left[1 - \xi^{(1)} \left(1 - \frac{\epsilon^{(1)}}{2} \right) \right] q^2, \quad (30)$$

$$b_3^2 = \left[\frac{2\epsilon^{(1)}}{j^{(i)}\theta^{(i)}} + \left(1 - \frac{\xi^{(1)}}{\delta^{(1)}} \right) \right] q^2. \quad (31)$$

We assume that b_1, b_2 and b_3 are positive. The general solutions for displacements and rotation functions in the lower half space ($y < 0$) are of the form

$$w^{(1)} = m^{(1)} L_2 \exp(-b_2 y) \exp[iq(x - ct)], \quad (32)$$

$$\phi_1^{(1)} = \sum_{k=1}^3 \lambda_k L_k \exp(-b_k y) \exp[iq(x - ct)], \quad (33)$$

$$\phi_2^{(1)} = \sum_{k=1}^3 L_k \exp(-b_k y) \exp[iq(x - ct)], \quad (34)$$

where L_1, L_2 and L_3 are constants,

$$M^{(1)} = \frac{j^{(i)} C_4^{(1)2} + C_5^{(1)2} (b_2^2 - q^2) - 2C^2 + \rho^{(1)} j^{(i)} q^2 c^2}{-C_3^{(1)2} iq}, \quad (35)$$

and

$$\lambda_1 = \frac{q}{ib_1}, \quad \lambda_2 = \frac{b_2}{iq}, \quad \lambda_3 = \frac{b_3}{iq}. \quad (36)$$

The system of equations in $A^{(2)}, B^{(2)}$ and $C^{(2)}$ are similar to those in $A^{(1)}, B^{(1)}$ and $C^{(1)}$ except for a change in superscript. Hence the solutions $w^{(2)}, \phi_1^{(2)}, \phi_2^{(2)}$ for the upper half space ($y > 0$) are

$$w^{(2)} = M^{(2)} L_5 \exp(b_5 y) \exp[iq(x - ct)], \quad (37)$$

$$\phi_1^{(2)} = \sum_{k=4}^6 \lambda_k L_k \exp(b_k y) \exp[iq(x - ct)], \quad (38)$$

$$\phi_2^{(2)} = \sum_{k=4}^6 L_k \exp(b_k y) \exp[iq(x - ct)], \quad (39)$$

where L_4, L_5 and L_6 are constants; b_4^2, b_5^2, b_6^2 are roots of the determinant of the coefficients of the equations in $A^{(2)}, B^{(2)}, C^{(2)}$ by neglecting $\epsilon^{(2)2}$,

$$M^{(2)} = \frac{j^{(2)} C_4^{(2)2} + C_5^{(2)2} (b_5^2 - q^2) - 2c^2 + \rho^{(2)} j^{(2)} q^2 c^2}{-C_3^{(2)2} iq}. \quad (40)$$

Further,

$$\lambda_4 = \frac{-q}{ib_4}, \quad \lambda_5 = \frac{-b_5}{iq}, \quad \lambda_6 = \frac{-b_6}{iq}, \quad (41)$$

and b_4^2, b_5^2, b_6^2 are respectively equal to the right-hand sides of equations (29) and (31) replacing superscript 1 by 2. On the stress free boundary surface the required continuous boundary conditions are

$$\begin{aligned} w^{(1)} &= w^{(2)}, & \phi_1^{(1)} &= \phi_1^{(2)}, & \phi_2^{(1)} &= \phi_2^{(2)}, \\ t_{23}^{(1)} &= t_{23}^{(2)}, & m_{21}^{(1)} &= m_{21}^{(2)}. \end{aligned} \quad (42)$$

These boundary conditions involve the macro-displacement and micro-rotations. Substituting equations (32) to (34) and (37) to (39) into equations (42) we get

$$M^{(i)} L_2 - M^{(2)} L_5 = 0, \quad (43)$$

$$\lambda_1 L_{14} + \lambda_2 L_2 + \lambda_3 L_3 - \lambda_4 L_4 - \lambda_5 L_5 - \lambda_6 L_6 = 0, \quad (44)$$

$$L_1 + L_2 + L_3 - L_4 - L_5 - L_6 = 0, \quad (45)$$

$$A_2^{(1)} b_2 m^{(1)} L_2 + A_2^{(2)} b_5 m^{(2)} L_5 = 0, \quad (46)$$

$$\begin{aligned} & \left[B_3^{(1)} iq - B_4^{(1)} \lambda_1 b_1 \right] L_1 + \left[B_3^{(1)} iq - B_4^{(1)} \lambda_2 b_2 \right] L_2 \\ & + \left[B_3^{(1)} iq - B_4^{(1)} \lambda_3 b_3 \right] L_3 - \left[B_3^{(2)} iq + B_4^{(2)} b_4 \lambda_4 \right] L_4 \\ & - \left[B_3^{(2)} iq + B_4^{(2)} \lambda_5 b_5 \right] L_5 - \left[B_3^{(2)} iq + B_4^{(2)} \lambda_6 b_6 \right] L_6 = 0, \end{aligned} \quad (47)$$

$$\begin{aligned} & \left[\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)} \right) b_1 - B_5^{(1)} \lambda_1 iq \right] L_1 \\ & + \left[\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)} \right) b_2 - B_5^{(1)} \lambda_2 iq \right] L_2 \\ & + \left[\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)} \right) b_3 - B_5^{(1)} \lambda_3 iq \right] L_3 \\ & + \left[\left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)} \right) b_4 + B_5^{(2)} \lambda_4 iq \right] L_4 \\ & + \left[\left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)} \right) b_5 + B_5^{(2)} \lambda_5 iq \right] L_5 \\ & + \left[\left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)} \right) b_6 + B_5^{(2)} \lambda_6 iq \right] L_6 = 0. \end{aligned} \quad (48)$$

A non-vanishing solution for L_1, L_2, L_3, L_4, L_5 and L_6 exists if and only if the determinant of the coefficients is zero:

$$|b_{ij}| = 0, \quad (49)$$

where

$$\begin{aligned} b_{11} &= 0, & b_{12} &= M^{(1)}, \\ b_{13} &= 0, & b_{14} &= 0, \\ b_{15} &= -M^{(2)}, & b_{16} &= 0; \\ b_{21} &= \lambda_1, & b_{22} &= \lambda_2, \\ b_{23} &= \lambda_3, \\ b_{24} &= -\lambda_4, & b_{25} &= -\lambda_5, \\ b_{26} &= -\lambda_6; \\ b_{31} &= 1, & b_{32} &= 1, & b_{33} &= 1, \\ b_{34} &= -1, & b_{35} &= -1, & b_{36} &= -1; \\ b_{41} &= 0, & b_{42} &= A_2^{(1)} b_2 M^{(1)}, & b_{43} &= 0, \\ b_{44} &= 0, & b_{45} &= A_2^{(2)} b_5 M^{(2)}, \\ b_{46} &= 0; \\ b_{51} &= B_3^{(1)} iq - B_4^{(1)} \lambda_1 b_1, \\ b_{52} &= B_3^{(1)} iq - B_4^{(1)} \lambda_2 b_2, \\ b_{53} &= B_3^{(1)} iq - B_4^{(1)} \lambda_3 b_3, \\ b_{54} &= -\left[B_3^{(2)} iq + B_4^{(2)} b_4 \lambda_4 \right], \end{aligned}$$

$$\begin{aligned}
b_{55} &= -\left[B_3^{(2)} i q + B_4^{(2)} \lambda_5 b_5\right], \\
b_{56} &= -\left[B_3^{(2)} i q + B_4^{(2)} \lambda_6 b_6\right]; \\
b_{61} &= \left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_1 - B_5^{(1)} \lambda_1 i q, \\
b_{62} &= \left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_2 - B_5^{(1)} \lambda_2 i q, \\
b_{63} &= \left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_3 + B_5^{(1)} \lambda_3 i q, \\
b_{64} &= \left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) b_4 + B_5^{(2)} \lambda_4 i q, \\
b_{65} &= \left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) b_5 + B_5^{(2)} \lambda_5 i q, \\
b_{66} &= \left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) b_6 + B_5^{(2)} \lambda_6 i q.
\end{aligned}$$

The determinant 49 can be expressed as two factors; hence each factor equals zero. Thus we have,

$$M^{(1)} M^{(2)} \left(A_2^{(2)} b_5 + A_2^{(1)} b_2 \right) = 0, \quad (50)$$

and

$$|c_{ij}| = 0 \quad (i, j = 1, 2, 3, 4), \quad (51)$$

where

$$\begin{aligned}
c_{11} &= \lambda_1, & c_{12} &= \lambda_3, & c_{13} &= -\lambda_4, & c_{14} &= -\lambda_6; \\
c_{21} &= 1, & c_{22} &= 1, & c_{23} &= -1, & c_{24} &= -1; \\
c_{31} &= b_{51}, & c_{32} &= b_{53}, & c_{33} &= b_{54}, & c_{34} &= b_{56}; \\
c_{41} &= b_{61}, & c_{42} &= b_{63}, & c_{43} &= b_{64}, & c_{44} &= b_{66}.
\end{aligned}$$

It is interesting to note that equation (50) gives two additional waves (depending only on micro-morphic constants) not encountered in classical elasticity. As equation (51) yields an equation in complex form, a further discussion on it is not initiated.

Now we study the effect of micro-strains in the present problem. We seek the plane harmonic solution of (18) to (20) as

$$\begin{aligned}
\phi_{(31)}^{(1)} &= N_1 \exp\left(-\ell^{(1)} y\right) \exp[iq(y - ct)], \\
\phi_{(32)}^{(1)} &= N_2 \exp\left(-\ell^{(1)} y\right) \exp[iq(y - ct)], \\
\phi_{(33)}^{(1)} &= N_3 \exp\left(-\ell^{(1)} y\right) \exp[iq(y - ct)],
\end{aligned} \quad (52)$$

and

$$\begin{aligned}
\phi_{31}^{(2)} &= N_4 \exp\left(-\ell^{(2)} y\right) \exp[iq(y - ct)], \\
\phi_{32}^{(2)} &= N_5 \exp\left(-\ell^{(2)} y\right) \exp[iq(y - ct)], \\
\phi_{33}^{(3)} &= N_6 \exp\left(-\ell^{(2)} y\right) \exp[iq(y - ct)],
\end{aligned} \quad (53)$$

$$\ell^{(1)2} = \frac{A_5^{(1)}}{B_2^{(1)}} + \left(1 - \frac{\rho^{(1)} j^{(1)} C^2}{4B_2^{(1)}}\right) q^2,$$

$$\ell^{(2)2} = \frac{A_5^{(2)}}{B_2^{(2)}} + \left(1 - \frac{\rho^{(2)} j^{(2)} C^2}{4B_2^{(2)}}\right) q^2,$$

and N_i ($i = 1, 2, 3, 4, 5, 6$) are constants. The boundary conditions to be satisfied involving the micro-strains are

$$\begin{aligned}
\phi_{31}^{(1)} &= \phi_{31}^{(2)}, & \phi_{32}^{(1)} &= \phi_{32}^{(2)}, & \phi_{33}^{(1)} &= \phi_{33}^{(2)}, \\
t_{2(32)}^{(1)} &= t_{2(32)}^{(2)}, & t_{2(33)}^{(1)} &= t_{2(33)}^{(2)}, & t_{2(31)}^{(1)} &= t_{2(31)}^{(2)}.
\end{aligned} \quad (54)$$

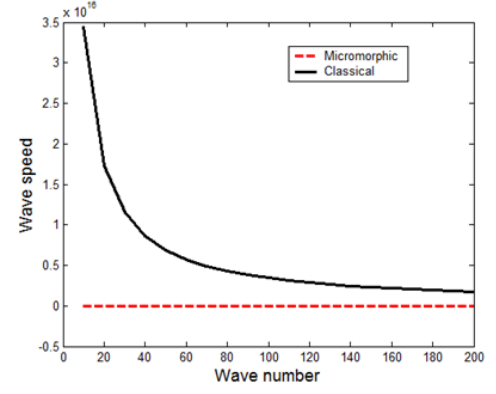


Figure 2: Wave number vs. wave speed.

Substituting equations (52) and (53) into equation (54) we obtain the frequency equation:

$$\frac{A_5^{(1)}}{B_2^{(1)}} + \left(1 - \frac{\rho^{(1)} j^{(1)} C^2}{4B_2^{(1)}}\right) q^2 - \frac{A_5^{(2)}}{B_2^{(2)}} - \left(1 - \frac{\rho^{(2)} j^{(2)} C^2}{4B_2^{(2)}}\right) q^2 = 0. \quad (55)$$

4. Numerical application

To study the wave speed at micro-morphic interfaces, the following relevant parameters of material-1 and material-2 are considered from Somaiah and Ravikumar [16] and Kumar et al. [17], respectively.

Material-1 (Aluminium epoxy) parameters:

$$\begin{aligned}
\lambda^{(1)} &= 7.59 \times 10^{10} \text{ N/m}^2, \\
\mu^{(1)} &= 1.89 \times 10^{10} \text{ N/m}^2, \\
K^{(1)} &= 0.0149 \times 10^{10} \text{ N/m}^2,
\end{aligned}$$

$$\begin{aligned}
j^{(1)} &= 0.000196 \text{ m}^2, \\
\rho^{(1)} &= 2190 \text{ kg/m}^3, \\
\sigma_2^{(1)} &= 0.0012 \times 10^3 \text{ N}, \\
\sigma_5^{(1)} &= 0.003 \times 10^3 \text{ N}.
\end{aligned}$$

Material-2:

$$\begin{aligned}
\lambda^{(2)} &= 9.4 \times 10^{10} \text{ N/m}^2, \\
\mu^{(2)} &= 4 \times 10^{10} \text{ N/m}^2, \\
K^{(2)} &= 0.125 \times 10^{10} \text{ N/m}^2,
\end{aligned}$$

$$j^{(2)} = 2 \times 10^{-2} \text{ m}^2, \quad (56)$$

$$\rho^{(2)} = 1.74 \times 10^3 \text{ kg/m}^3, \quad (57)$$

$$\sigma_2^{(2)} = 0.04 \times 10^3 \text{ N (randomly)}, \quad (58)$$

$$\sigma_5^{(2)} = 0.02 \times 10^3 \text{ N (randomly)}. \quad (59)$$

Figure 2 delineates that the wave speed at micro-morphic contact half-space boundaries is nearly constant, while at classical elastic solid contact half-space boundaries it rapidly decreases in the range of wave number 0 to 100 and is constant at wave numbers greater than 100. The speed of the wave at classical elastic solid

contact half-space boundaries reaches a maximum value more than 3.5×10^{16} m/sec, and a minimum value of 0.5×10^{16} m/sec at low and high wave numbers respectively. It is also observed that the wave speed at classical elastic solid contact half-space boundaries is more than that of micro-morphic contact half-space boundaries. Due to small wave velocity, micro-morphic contact interfaces are safer than classical elastic solid contact interfaces in earthquake regions.

5. Conclusion

To study the SH-wave propagation at the interface boundaries of two micro-morphic elastic solid half-spaces, the governing equations are converted into the xy -plane and solved with the method of plane harmonic solution. The secular equations of SH-waves are derived analytically. Hence, they are independent of classical results; it is an additional wave found and it is dispersive. If two media have the same mechanical properties this equation is identically satisfied. So, no wave exists corresponding to micro-strain when both media have the same mechanical properties such as constants, density and micro inertia. With the use of a MATLAB program for a particular model, the following concluding remarks are made:

- i The wave speed at classical elastic solid contact half-space boundaries is more than that of micro-morphic contact half-space boundaries.
- ii The wave speed at micro-morphic contact half-space boundaries is constant, while at classical elastic solid contact half-space boundaries it rapidly decreases in the small wave number range and is constant at large wave numbers.
- iii The speed of the wave at classical elastic solid contact half-space boundaries reaches a maximum value and a minimum value.

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