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Chatterjea fixed point theorem in rectangular b-metric space

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Abstract

In this paper, a Chatterjea-type fixed point theorem in a rectangular b-metric space is proved, thereby providing a complete solution to an open problem posed by Reny George, S. Radenovic, K.P. Reshma and S. Shukla (Rectangular b-metric space and contraction principles).

Keywords: Fixed point; Metric space; b-metric space; Rectangular b-metric space

1. Introduction

Fixed point theory is one of the most important tools in nonlinear functional analysis and applied mathematics. Since Banach's contraction principle, numerous generalizations have emerged. For instance, Chatterjea [1] initiated the study of fixed points under a novel contraction condition.

The main result of [1] is the following theorem.

Theorem 1. *Let (X,d) be a complete metric space and let $T : X \to X$ be a mapping such that there exists a number $h \in \left[0, \frac{1}{2}\right)$ such that*

$$d(Tx,Ty) \le h[d(x,Ty) + d(y,Tx)]$$

for all $x, y \in X$. Then, T has a unique fixed point.

Subsequent work by E. Karapinar and H. K.Nashine [2] , C.B.Ampadu and Boateng [3], Gautam et al. [4], Nazam et al. [5], O.Popescu [6], and others [7, 8, 9, 10] explored this condition in depth. Meanwhile, generalizations of Banach's principle in various metric frameworks have also been studied—Branciari [10] introduced rectangular metric spaces, Bakhtin [11] studied b-metric spaces, and George et al.[12] combined these ideas into b-rectangular metric spaces, raising several open questions. Some other fixed-point theorems in b-rectangular metric spaces can be seen [13, 14]

In 2018, Mitrović [15] relaxed the contraction coefficient in the b-rectangular metric and posed an open question concerning a Chatterjea-type theorem. In this article, We address this question by proving a Chatterjea-type fixed point theorem in b-rectangular metric spaces, thereby extending and unifying several earlier results.

2. Preliminaries

In the following section, the definitions that are fundamental to our subsequent analysis are compiled.

2.1. Metric space

The concept of a metric space was axiomatically formulated by Maurice René Fréchet [16] in 1906.

Definition ([16]). *Let X be a non-empty set and let $d : X \times X \to \mathbb{R}^+$ be a function satisfying the conditions:*

M1:	$d(x,y) \ge 0$	*(non-negative)*
M2:	$d(x,y) = 0 \iff x = y$	*(indiscernibles)*
M3:	$d(x,y) = d(y,x)$	*(symmetry)*
M4:	$d(x,y) \le d(x,z) + d(z,y)$	*(triangle inequality)*

for all $x, y, z \in X$. Then d is called a metric on X and the pair (X,d) is called metric space.

2.2. Definitions of Convergence sequence, Cauchy sequence and Complete matric space

Convergent sequence [17]: A sequence $\{x_n\}$ in a metric space (X,d) is said to converge to x in X if

$$\lim_{n\to\infty} d(x_n,x) = 0 = \lim_{n\to\infty} d(x,x_n)$$

we denote this by

$$\lim_{n\to\infty} x_n = x$$

Cauchy sequence [17]: A sequence $\{x_n\}$ in a metric space (X,d) is said to be Cauchy sequence if for a given $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $m, n > n_0$, we have $d(x_n, x_m) < \varepsilon$ or $d(x_m, x_n) < \varepsilon$.

Complete metric space [17]: A metric space (X,d) is said to be complete if every Cauchy sequence in X is convergent to a point in X.

2.3. b-metric space

In 1989, Bakhtin [11] introduced the concept of a b-metric space, which generalizes the notion of a metric space by relaxing the triangle inequality.

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Definition ([11, 18]). *Let X be a non-empty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{R}^+$ is called b-metric on X if the following conditions hold: For all $x, y, z \in X$*

1. $d(x, y) = 0 \iff x = y$
2. $d(x, y) = d(y, x)$
3. $d(x, z) \leq s[d(x, y) + d(y, z)]$

The pair (X, d) is called b-metric space, in short bMS.

2.4. Rectangular Metric Space

In 2000, Branciari [10] introduced the notion of rectangular metric space as follows:

Definition ([10]). *Let X be a non-empty set and a function $d : X \times X \to \mathbb{R}^+$ be a mapping such that for all $x, y \in X$, and for all distinct points $u, v \in X$, each of them different form x and y, one has*

1. $d(x, y) = 0 \iff x = y$
2. $d(x, y) = d(y, z)$
3. $d(x, y) \leq d(x, u) + d(u, v) + d(v, y)$ *(Rectangular inequality)*

Then d is called a rectangular metric (or generalized metric) on X and the pair (X, d) is a rectangular metric space or generalized metric space. (Shortly RMS or gms respectively.)

2.5. Rectangular b-Metric Space

In 2015, George et al. [12] introduced the notion of rectangular b-metric space, which is given as follows:

Definition ([12]). *Let X be a non-empty set and the mapping $d : X \times X \to [0, \infty)$ satisfies:*

$$\begin{aligned} &(RbM1) \quad d(x, y) = 0 \text{ if and only if } x = y \\ &(RbM2) \quad d(x, y) = d(y, x) \quad \text{for all } x.y \in X \\ &(RbM3) \quad \text{there exists a real number } s \geq 1 \text{ such that} \\ &\qquad\qquad d(x, y) \leq s[d(x, u) + d(u, v) + d(v, y)] \\ &\qquad\qquad \text{(Rectangular inequality)} \end{aligned}$$

for all $x, y \in X$ and all distinct pints $u, v \in X$ and each of them different from x and y. Then d is called a rectangular b-metric on X and the pair (X, d) is a rectangular b-metric space (in short $RbMS$) and with coefficient s.

2.6. Relationships Between Metric, b-Metric space, Rectangular Metric, and rectangular b- Metric Spaces

Every metric space is a rectangular metric space (generalized metric space, g.m.s.), and every rectangular metric space is a rectangular b-metric space with coefficient $s = 1$. However, the converse is not necessarily true (see example 1).

Furthermore, every metric space is a b-metric space, and every b-metric space is a rectangular b-metric space. Additionally, every b-metric space with coefficient s is a rectangular b-metric space with coefficient s^2, but the converse is not necessarily true (see example 1).

Thus, we have the following diagram.

$$\begin{tikzcd} \boxed{\text{Metric space}} \arrow[r] \arrow[d] & \boxed{b\text{-Metric space}} \arrow[d] \\ \boxed{\text{Rectangular metric space}} \arrow[r] & \boxed{\text{Rectangular } b\text{-metric space}} \end{tikzcd}$$

where arrows stand for inclusions. The inverse inclusions do not hold.

Limitation of the converse statement:

While it is established that every metric space is a rectangular metric space and every rectangular metric space is a rectangular b-metric space (with coefficient $s = 1$), the converses of these statements do not always hold. That is, there exist rectangular b-metric spaces that are not rectangular metric spaces, and b-metric space that are not metric spaces. Similarly, although every b-metric space with coefficient s is a rectangular b-metric space with coefficient s^2, the reverse implication does not necessarily hold. This means that there exist rectangular b-metric spaces with coefficient s^2 that do not satisfy the conditions of a b-metric space with coefficient s.

Example 1. *[12] Let $X = A \cup B$, where $A = \{\frac{1}{n} : n \in \mathbb{N}\}$ and B is the set of all positive integers. We define $d : X \times X \to [0, \infty)$ such that $d(x, y) = d(y, x)$ for all $x, y \in X$ and*

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 2\alpha & \text{if } x, y \in A \\ \frac{\alpha}{2n} & \text{if } x \in A \text{ and } y \in \{2, 3\} \\ \alpha & \text{otherwise} \end{cases}$$

where $\alpha > 0$ is a constant. Then $(X,\ d)$ is a rectangular b metric space with coefficient, $s = 2 > 1$. But converse is not true.

2.7. Fixed Point Theory and Contractions

Fixed point [19]: A point $x \in X$ is a fixed point of a mapping $f : X \to X$ if $f(x) = x$.

Lipschitz Mapping [20]: A mapping f is a Lipschitz mapping if there exists $k \geq 0$ such that

$$d(f(x), f(y)) \leq kd(x, y) \text{ for all } x, y \in X$$

Banach Contraction Principle: [21] If X is a complete metric space and $T : X \to X$ is a contraction mapping (i.e. there exists $k \in [0, 1)$ such that $d(Tx, Ty) \leq kd(x, y)$), then T has a unique fixed point x_0 in X, and for each $x \in X$, the sequence $T^n(x)$ converges to x_0.

3. Main Theorem

The following theorem provides an analogue of Chatterjeea's contraction principle within the framework of rectangular b-metric space.

Theorem 2. *Let (X, d) be a complete rectangular b-metric space with $s > 1$ and $T : X \to X$ be a mapping satisfying*

$$d(Tx, Ty) \leq k\,[d(x, Ty) + d(y, Tx)] \tag{1}$$

for all $x, y \in X$, where $k \in \left[0, \dfrac{1}{s+1}\right]$. Then T has a unique fixed point.

Proof. Let x_0 be arbitrary point in X. We define a sequence $\{x_n\}$ in X by

$$x_{n+1} = Tx_n = T^{n+1}x_0$$

for all $n \geq 0$. We have to show that $\{x_n\}$ is Cauchy sequence. If $x_{n+1} = x_n$, then x_n is fixed point of T.

So, Assume that $x_n \neq x_{n+1}$ for all $n \geq 0$. Setting $d(x_n, x_{n+1}) = d_n$, it follows from (1) that

$$\begin{aligned} d(x_n, x_{n+1}) &= d(Tx_{n-1}, Tx_n) \\ &\leq k\,[d(x_{n-1}, Tx_n) + d(x_n, Tx_{n-1})] \\ &= k\,[d(x_{n-1}, x_{n+1}) + d(x_n, x_n)] \\ &\leq k\,[d(x_{n-1}, x_n) + d(x_n, x_{n+1})] \end{aligned}$$

So,

$$\begin{aligned} d(x_n, x_{n+1}) &\leq \frac{k}{1-k}\, d(x_{n-1}, x_n) \\ &= \beta\, d(x_{n-1}, x_n) \end{aligned}$$

where, $\beta = \dfrac{k}{1-k} < \dfrac{1}{s}$ $\quad \left(\text{as } k < \dfrac{1}{1+s} \right)$

Repeating this process, we obtain

$$d(x_n, x_{n+1}) \leq \beta^n\, d(x_0, x_1)$$

$$\text{i.e. } d_n \leq \beta^n\, d_0 \tag{2}$$

Also, we can assume that x_0 is not a periodic point of T. Indeed if $x_0 = x_n$. Then using (2) for any $n \geq 2$. We have,

$$\begin{aligned} d(x_0, Tx_0) &= d(x_n, Tx_n) \\ d(x_0, x_1) &= d(x_n, x_{n+1}) \\ d_0 &= d_n \leq \beta^n\, d_0 \end{aligned}$$

a contradiction. Therefore we must have $d_0 = 0$ i.e. $x_0 = x_1$ and so x_0 is a fixed point of T.

Now, we assume that $x_n \neq x_m$ for all distinct $n, m \in \mathbb{N}$. Again using (1) and (2), for any $n \in \mathbb{N}$. We get,

$$\begin{aligned} d(x_n, x_{n+2}) &= d(Tx_{n-1}, Tx_{n+1}) \\ &\leq k\,[d(x_{n-1}, Tx_{n+1}) + d(x_{n+1}, Tx_{n-1})] \\ &\leq k\,[d(x_{n-1}, x_{n+2}) + d(x_{n+1}, x_n)] \\ &\leq k\,[d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) \\ &\quad + d(x_{n+1}, x_n)] \\ &= k\,[d_{n-1} + d_n + d_{n+1} + d_n] \\ &= k\,[d_{n-1} + 2d_n + d_{n+1}] \\ &\leq k\,[\beta^{n-1}\, d_0 + 2\beta^n\, d_0 + \beta^{n+1}\, d_0] \\ &= k\,\beta^{n-1}\,[1 + 2\beta + \beta^2]\, d_0 \\ &= \alpha\,\beta^{n-1}\, d_0 \end{aligned}$$

where $\alpha = k\,(1+\beta)^2 > 0$

Therefore

$$d(x_n, x_{n+2}) \leq \alpha\,\beta^{n-1}\, d_0 \tag{3}$$

For the sequence $\{x_n\}$, we consider

$$d(x_n, x_{n+p}) \text{ in two cases}$$

If p is odd say $2m+1$, Then using (2), we obtain

$$\begin{aligned} &d(x_n, x_{n+2m+1}) \\ &\leq s\,[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) \\ &\quad + d(x_{n+2}, x_{n+2m+1})] \\ &\leq s\,[d_n + d_{n+1}] + s^2\,[d(x_{n+2}, x_{n+3}) \\ &\quad + d(x_{n+3}, x_{n+4}) + d(x_{n+4}, x_{n+2m+1})] \\ &\leq s\,[d_n + d_{n+1}] + s^2\,[d_{n+2} + d_{n+3}] \\ &\quad + s^3\,[d_{n+4} + d_{n+5}] + \cdots + s^m\, d_{n+2m-2} \\ &\quad + s^m\, d_{n+2m-1} + s^m\, d(x_{n+2m}, x_{n+2m+1}) \\ &\leq s\,[\beta^n\, d_0 + \beta^{n+1}\, d_0] \\ &\quad + s^2\,[\beta^{n+2}\, d_0 + \beta^{n+3}\, d_0] \\ &\quad + s^3\,[\beta^{n+4}\, d_0 + \beta^{n+5}\, d_0] \\ &\quad + \cdots + s^m\, \beta^{n+2m-2}\, d_0 \\ &\quad + s^m\, \beta^{n+2m-1}\, d_0 + s^m\, d_{n+2m} \\ &= s\,\beta^n\,[1 + s\,\beta^2 + s^2\,\beta^4 + \cdots \\ &\quad + s^{m-1}\,\beta^{2(m-1)}]d_0 \\ &\quad + s\,\beta^{n+1}[1 + s\beta^2 + s^2\,\beta^4 + \cdots \\ &\quad + s^{m-1}\,\beta^{2(m-1)}]d_0 \\ &\quad + s^m\,\beta^{n+2m}\, d_0 \\ &= s\beta^n \left[\frac{1 - (s\beta^2)^{m-1}}{1 - s\beta^2} \right] (1+\beta) d_0 \\ &\quad + (s\beta)^m \beta^{n+m} d_0 \\ &\leq \frac{1+\beta}{1 - s\beta^2}\, s\beta^n d_0 + \beta^{n+m} d_0 \\ &\quad \left(\because s\beta^2 < 1 \text{ and } \beta < \frac{1}{s} \right) \end{aligned}$$

Therefore,

$$d(x_n, x_{n+2m+1}) \leq \frac{1+\beta}{1 - s\beta^2}\, s\beta^n d_0 + \beta^{n+m} d_0 \tag{4}$$

If p is even say $2m$. Then, using (2) and (3), we obtain

$$\begin{aligned} d(x_n, x_{n+2m}) &\leq s\,[d(x_n, x_{n+1}) \\ &\quad + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_{n+2m})] \\ &\leq s\,[d_n + d_{n+1}] + s^2\,[d(x_{n+2}, x_{n+3}) \\ &\quad + d(x_{n+3}, x_{n+4}) + d(x_{n+4}, x_{n+2m})] \\ &\leq s\,[d_n + d_{n+1}] + s^2\,[d_{n+2} + d_{n+3}] \\ &\quad + s^3\,[d_{n+4} + d_{n+5}] + \cdots \\ &\quad \cdots + s^{m-1}\,[d_{n+2m-4} + d_{n+2m-3}] \\ &\quad + s^{m-1}\, d(x_{n+2m-2}, x_{n+2m}) \\ &\leq s\,[\beta^n\, d_0 + \beta^{n+1}\, d_0] + s^2\,[\beta^{n+2}\, d_0 + \beta^{n+3}\, d_0] \\ &\quad + s^3\,[\beta^{n+4}\, d_0 + \beta^{n+5}\, d_0] + \cdots \\ &\quad + s^{m-1}\,[\beta^{n+2m-4}\, d_0 + \beta^{n+2m-3}\, d_0] \\ &\quad + s^{m-1}\,\alpha\beta^{n+2m-3} d_0 \\ &= s\,\beta^n\,[1 + s\,\beta^2 + s^2\,\beta^4 + \cdots]d_0 \\ &\quad + s\,\beta^{n+1}[1 + s\beta^2 + s^2\,\beta^4 + \cdots]d_0 \\ &\quad + s^{m-1}\,\alpha\beta^{n+2m-3}\, d_0 \\ &\leq \frac{1+\beta}{1 - s\beta^2}\, s\beta^n d_0 + s^{m-1}\alpha\beta^{n+2m-3} d_0 \end{aligned}$$

So,

$$\begin{aligned} d(x_n, x_{n+2m}) &\leq \frac{1+\beta}{1 - s\beta^2}\, s\beta^n d_0 + s^{m-1}\alpha\beta^{n+2m-3} d_0 \\ &< \frac{1+\beta}{1 - s\beta^2}\, s\beta^n d_0 + \alpha(s\beta)^{2m}\beta^{n-3} d_0 \\ &\quad (\quad \text{as } s > 1) \\ &\leq \frac{1+\beta}{1 - s\beta^2}\, s\beta^n d_0 + \alpha\beta^{n-3} d_0 \;(\text{ as } \beta \leq \frac{1}{s}) \end{aligned}$$

Therefore,

$$d(x_n, x_{n+2m}) \leqq \frac{1+\beta}{1 - s\beta^2}\, s\beta^n d_0 + \alpha\beta^{n-3} d_0 \tag{5}$$

It follows from (3) and (4) that

$$\lim_{n\to\infty} d(x_n, x_{n+p}) = 0 \text{ for all } p > 0 \tag{6}$$

Thus the sequence $\{x_n\}$ is a Cauchy sequence in X.

By completeness of (X, d), There exists $u \in X$ such that

$$\lim_{n\to\infty} x_n = u \tag{7}$$

We have to show that u is a fixed point of T. Again for any $n \in \mathbb{N}$, we have,

$$\begin{aligned}
d(u, Tu) &\leq s[d(u, x_n) + d(x_n, x_{n+1}) + d(x_{n+1}, Tu)] \\
&= s[d(u, x_n) + d_n + d(Tx_n, Tu)] \\
&\leq s[d(u, x_n) + d_n + k[d(x_n, Tu) + d(u, Tx_n)]] \\
&= s[d(u, x_n) + d_n + kd(x_n, Tu) + kd(u, x_{n+1})] \\
&\leq s[d(u, x_n) + \beta^n d_0 + kd(x_n, Tu) + kd(u, x_{n+1})]
\end{aligned}$$

using (6), (7) and the fact that $k < \dfrac{1}{1+s}$, it follows from the above inequality that

$$d(u, Tu) \leq s\, k\, d(u, Tu)$$

It must be $d(u, Tu) = 0$ i.e. $Tu = u$

Thus u is a fixed point of T.

Uniqueness: Let v be another fixed point of T in X then $Tv = v$. Assume that $u \neq v$,Then it follows from contraction that

$$\begin{aligned}
d(u, v) &= d(Tu, Tv) \\
&\leq k[d(u, Tv) + d(v, du)] \\
&= k[d(u, v) + d(v, u)] \\
&= 2k\, d(u, v)
\end{aligned}$$

The above inequality is possible only if

$$d(u, v) = 0 \implies u = v$$

Hence, fixed point is unique. □

The following is an example of Theorem 2.

Example 2. *Let $X = \mathbb{R}$ be the set of real numbers and define the distance function $d : X \times X \to [0, \infty)$ by*

$$d(x, y) = \begin{cases} |x - y| & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

Define a mapping $T : X \to X$ defined by $T(x) = \frac{x}{2}$

Then

1. *(X, d) is a rectangular b-metric space with coefficient $s = 1$.*
2. *T satisfies the Chatterjea contraction condition with $k = \frac{1}{3}$, where $x = 2$ & $y = 0$*
3. *T has unique fixed point $x = 0$*
4. *Let point $x_0 \in X$, define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$. Then $\{x_n\}$ converges to the fixed point of T.*

4. Conclusions

In this paper, a Chatterjea-type fixed point theorem is proved in rectangular b-metric spaces, addressing an open question raised by Reny George, S. Radenovic, K. P. Reshma, and S. Shukla. Several existing results in fixed point theory are generalized and unified by this result.

References

[1] Chatterjea S K, Fixed-point theorems, *Dokladi na Bolgarskata Akademiya na Naukite*, 25(6) (1972) 727–+.

[2] Karapınar E & Nashine H K, Fixed point theorem for cyclic chatterjea type contractions, *Journal of Applied Mathematics*, 2012(1) (2012) 165698. https://doi.org/10.1155/2012/165698.

[3] Ampadu C B, Chatterjea-type contraction mapping theorem for four self-mappings in cone pentagonal metric space, *Earthline Journal of Mathematical Sciences*, 15(2) (2025) 169–180. https://doi.org/10.34198/ejms.15225.169180.

[4] Gautam P, Mishra V N, Ali R & Verma S, Interpolative chatterjea and cyclic chatterjea contraction on quasi-partial b-metric space, *AIMS Mathematics*, 6(2) (2021) 1727–1742. https://doi.org/10.3934/math.2021103.

[5] Nazam M, Muhammad A & Hussain A, Fixed point theorems for chatterjea type f-contraction on closed ball, *Journal of Analysis and Number Theory*, 5. https://doi.org/10.18576/jant/050105.

[6] Păcurar C & Popescu O, Fixed point theorem for generalized chatterjea type mappings, *Acta Mathematica Hungarica*, 173(2) (2024) 500–509. https://doi.org/10.1007/s10474-024-01455-6.

[7] Dhungana C, Bajracharya K, Pahari N P & Ojha D, On a generalization of chatterjee's fixed point theorem inb-metric space, *Nepal Journal of Mathematical Sciences*, 4(2). https://doi.org/10.3126/njmathsci.v4i2.59526.

[8] Islamiyah N I, Zakir M & Nurwahyu B, Common fixed-point theorems of generalization of kannan, chatterjea, and reich contractive on b-metric space with an application, *Baghdad Science Journal*, 21(9) (2024) 2984–2984. https://doi.org/10.21123/bsj.2024.9198.

[9] Gayathri R O & Hemavathy R, Chatterjea fixed point theorem in rectangular b-metric space endowed with graph, *Aligarh Bulletin of Mathematics*, 39(2) (2020) 27–34. ISSN 0304-9787. URL http://www.tabmaths.com/issues/Tabmath-Vol39-Number-2-2021/3-gayathari.pdf.

[10] Branciari A, A fixed point theorem of banach-caccioppoli type on a class of generalized metric spaces, *Publ. Math. Debrecen*, 57(1-2) (2000) 31–37. https://doi.org/10.5486/PMD.2000.2133.

[11] Bakhtin I, The contraction mapping principle in quasimetric spaces, *Functional analysis*, 30 (1989) 26–37.

[12] George R, Radenovic S, Reshma K & Shukla S, Rectangular b-metric space and contraction principles, *Journal of Nonlinear Sciences and Applications*, 8(6) (2015) 1005–1013. https://doi.org/10.22436/jnsa.008.06.11.

[13] Ding H S, Öztürk V & Radenovic S, On some new fixed point results in b-rectangular metric spaces, *Journal of Nonlinear Sciences and Applications*, 8(4). https://doi.org/10.22436/jnsa.010.04.39.

[14] Zheng D, Wang P & Citakovic N, Meir-keeler theorem in b-rectangular metric spaces, *Journal of Nonlinear Sciences and Applications*, 10(4) (2017) 1786–1790. https://doi.org/10.22436/jnsa.010.04.39.

[15] Mitrovic Z D, George R & Hussain N, Some remarks on contraction mappings in rectangular b-metric spaces, *Bol. da Socie Paranaense de Matematica (3s)*, 22(2) (2018) 1–9. https://doi.org/10.5269/bspm.41754.

[16] Fréchet M, Sur quelques points du calcul fonctionnel, *Rendiconti del Circolo Matematico di Palermo*, 22(1) (1906) 1–72. ISSN 0009-725X. https://doi.org/10.1007/BF03018603.

[17] Simmons G F. *Introduction to topology and modern analysis*, vol. 1. McGraw-Hill New York (1963). URL https://ia600808.us.archive.org/0/items/in.ernet.dli.2015.141658/2015.141658.Introduction-To-Topology-And-Modern-Analysis.pdf.

[18] Czerwik S, Contraction mappings in b-metric spaces, *Acta mathematica et informatica universitatis ostraviensis*, 1(1) (1993) 5–11. URL http://dml.cz/dmlcz/120469.

[19] O'Searcoid M. *Metric spaces*. Springer Science & Business Media (2006). https://doi.org/10.1007/978-1-84628-627-8.

[20] Hunter J K & Nachtergaele B. *Applied analysis*. World Scientific Publishing Company (2001). URL https://www.math.ucdavis.edu/~hunter/book/pdfbook.html.

[21] Goebel K & Kirk W A. *Topics in metric fixed point theory*, vol. 28. Cambridge university press (1990). URL https://books.google.com.np/books?hl=en&lr=&id=_IVYz4p0BaYC&oi=fnd&pg=PP7&dq=K.+Goebel,+œTopics+in+metric+fixed+point+theory,+Cambridge+Studies+in+Advanced+Mathematics/Cambridge+University+Press,+vol.+28,+1990.&ots=6PxNrgKQvF&sig=RfXMk29qyfy-1f_-yJ5sBrQKNYM&redir_esc=y#v=onepage&q&f=false.