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A common fixed point theorem in fuzzy b-metric space using compatible mappings of type (K)

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Abstract

Our primary goal is to use the notion of compatible mappings of type (K) in complete fuzzy b-metric space to introduce a common fixed point theorem. Numerous earlier, comparable findings in the literature are enhanced and expanded upon by our generalization. The major consequence is presented with an illustrative example.

Keywords: Fuzzy metric space; Compatible mappings; Fixed point.

1. Introduction

One of the important generalization of metric space is b-metric space introduced by Bourbaki and Bakhtin [1] in 1989. In 1993, Czerwik [2] introduced an axiom which was more flexible than the triangular inequality and precisely defined a b-metric space with the view of generalizing the Banach contraction mapping theorem. These ideas lead Banach [3] to prove the well known fixed point theorem in 1922. This result is taken as a mile stone for the researchers to introduce the new results in the fixed point theory.

As the concept of fuzzy sets was introduced by Zadeh [4] in 1965. The fuzzy metric is used to measure the degree of nearness between any two objects with respect to the given parameter, was first introduced by Michalek and Kramosoil [5] in 1975. In 1988, Grabiec [6] using the idea of fuzzy metric space, proved the Banach fixed point theorem. In 1994, George and Veermani [7] modified the definition of fuzzy metric space given by Michalek and Kramosil [5] and defined the Hausdorff topology of fuzzy metric spaces. In 2015, Hussain et.al [8] introduced the idea of fuzzy b-metric space and established a relation between the parametric b-metric and fuzzy b-metric. Nadaban [9] modified the idea of fuzzy b-metric space established by Hussian in 2016. After this many researches have explored the different ideas in their research works on this domain. Mani et al. [10], introduced fixed point theorems in fuzzy b-metric spaces using two different t-norms. Bhandari et. al. introduced some come fixed point theorems in fuzzy b-metric space using the integrals as application in 2025 [11].

In 1986, Jungck [12] introduced the notion of compatible mappings, which are more general than commuting and weakly commuting mappings. In 1998, Cho et al. [13] introduced the concept of compatible mapping in fuzzy metric space. Different type of compatible mappings are introduced by many researchers with the follow of time. In 2014, Jha et al. [14] introduced the new notion of compatible mappings of type (K) in metric space. Manandhar et al.[15] extended it into fuzzy metric space and established some common fixed point theorems for the pairs of compatible mappings of type(K). Similarly in 2024, Bhandari et al. [16] introduced the fixed point theorems in fuzzy b-metric space using the compatible mapping of type (A).

Our objective is to establish a common fixed point theorem in complete fuzzy b-metric space under the new concept of compatible mapping of type (K), and to verify this result with an example. Our result extends and generalizes the result of K. Jha, V. Popa and K.B. Manandher [14].

As the consequence of our main finding, we obtained a property applying the minimum t-norm.

2. Preliminaries

Definition 1. [17] A mapping $* : I \times I \to I$, where $I = [0,1]$ is called a continuous triangular norm (t-norm) if it satisfies the following properties:

(i) Symmetry: $x * y = y * x$, for $x, y \in I$;

(ii) Monotonocity: $x * y \leq z * w$ whenever $x \leq z$ and $y \leq w$;

(iii) Associativity: $(x * y) * z = x * (y * z)$, where $x, y, z \in I$;

(iv) Boundary condition: $1 * x = x$, for all $x \in I$.

The idea of fuzzy metric space was first suggested in 1975 by Kramosil and Michalek [5] using the concept of continuous t-norm.

Definition 2. [5] Let W be an arbitrary set, $\mathbb{A}$ be a fuzzy set defined on $W \times W \times [0, \infty)$ and $*$ be a continuous t-norm. Then the order tuple $(W, \mathbb{A}, *)$ is said to be a fuzzy metric space satisfying the following properties, $\forall\, p, q, r \in W$ and $a, b > 0$

(i) $\mathbb{A}(p, q, 0) = 0$;

(ii) $\mathbb{A}(p, q, a) = 1$ for all $a > 0$ iff $p = q$

(iii) $\mathbb{A}(p, q, a) = \mathbb{A}(q, p, a)$;

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(iv) $\mathbb{A}(p,q,a) * \mathbb{A}(q,r,b) \leq \mathbb{A}(p,r,a+b)$ for all $a,b>0$;

(v) $\mathbb{A}(p,q,.) : (0,\infty) \to I$ is continuous from left,

where the expression $\mathbb{A}(p,q,a)$ represents the degree of closeness between p and q depending upon the parameter $a>0$.

This idea is modified by George and Veeramani in 1994 and defined as follow.

Definition 3. [7] The order tuple $(W,\mathbb{A},*)$ is said to be a fuzzy metric space if W is an arbitrary set, $*$ is a continuous t-norm and $\mathbb{A}$ is a fuzzy set defined on $W\times W\times[0,\infty)$ satisfying the following conditions, for all $p,q,r\in W$ and $a,b>0$

(i) $\mathbb{A}(p,q,a)>0$;

(ii) $\mathbb{A}(p,q,a)=1, a>0$ iif $p=q$;

(iii) $\mathbb{A}(p,q,a)=\mathbb{A}(q,p,a)$;

(iv) $\mathbb{A}(p,q,a) * \mathbb{A}(q,r,b) \leq \mathbb{A}(p,r,a+b)$ for all $a,b>0$;

(v) $\mathbb{A}(p,q,.) : (0,\infty) \to I$ is continuous.

Example 4. [18]Let $W=\mathbb{R}$, the set of all real numbers and t-norm is defined in terms of the product $x*y=x.y\,\forall\, x,y\in I$ and $p,q\in W, a>0$ defined as:

$$\mathbb{A}(p,q,a)=\begin{cases}\dfrac{a}{a+|p-q|} & \text{if } p,q\in W, a>0,\\ 0 & \text{if } p,q\in W, a=0.\end{cases}$$

Then $\mathbb{A}$ is a fuzzy metric on $\mathbb{R}$.

Definition 5. [9] Let W be an arbitrary set, $\mathbb{A}$ be a fuzzy set defined on $W\times W\times[0,\infty)$ where $*$ be a continuous t-norm and given $s\geq 1$. Then the order tuple $(W,\mathbb{A},*)$ is said to be a fuzzy b-metric space satisfying the following properties, $\forall\, p,q,r\in W$ and $a,b>0$

(i) $\mathbb{A}(p,q,a)>0$;

(ii) $\mathbb{A}(p,q,a)=1\,\forall\, a>0$ iif $p=q$;

(iii) $\mathbb{A}(p,q,a)=\mathbb{A}(q,p,a)$;

(iv) $\mathbb{A}(p,q,a) * \mathbb{A}(q,r,b) \leq \mathbb{A}(p,r,s(a+b))\,\forall\, a,b>0$;

(v) $\mathbb{A}(p,q,.) : (0,\infty) \to I$ is continuous;

(vi) $\lim_{a\to\infty}\mathbb{A}(p,q,a)=1$

Example 6. [9]Let $\mathbb{A}(p,q,a)=e^{\frac{-d(p,q)}{a^2}}$ where d is a b-metric on W, and t-norm is defined in terms of the product. Then it is a fuzzy b-metric space.

Definition 7. [6] Let $(W,\mathbb{A},*)$ be a fuzzy b-metric space. If we take a sequence $\{p_n\}$ from W then it is called to be convergent in $p\in W$ if

$\lim_{n\to\infty}\mathbb{A}(p_n,p,a)=1$ for each $a>0$.

Definition 8. [6] A sequence $\{p_n\}$ in W is called to be Cauchy sequence in W if $\lim_{n\to\infty}\mathbb{A}(p_n,p_{m+n},a)=1$ where $a>0$ and $m,n\geq n'$ where $n'\in N$.

A fuzzy b-metric space is said to be complete if every Cauchy sequence in it is convergent to the same point.

Definition 9. [13] Two self mappings f and g defined on a fuzzy b-metric space $(W,\mathbb{A},*)$ where $s\geq 1$ are said to be compatible

(a) if, for all $a>0$, $\lim_{n\to\infty}\mathbb{A}(fgp_n,gfp_n,a)=1$ where $\{p_n\}\in W$ which gives, $\lim_{n\to\infty}fp_n=\lim_{n\to\infty}gp_n=b$, where $b\in W$.

(b) They are said to be weakly compatible if they commute at their coincidence point, that is $fp=gp$ which gives that $fgp=gfp$.

(c) are said to be semi-compatible if, for all $a>0$, $\lim_{n\to\infty}\mathbb{A}(fgp_n,gp_n,a)=1$ where $\{p_n\}\in W$ with the property, $\lim_{n\to\infty}fp_n=\lim_{n\to\infty}gp_n=q$, where $q\in W$.

Definition 10. [15] The two self-mappings f and g defined on the fuzzy b-metric space $(W,\mathbb{A},*)$ are said to be compatible of type (K) if $\lim_{n\to\infty}\mathbb{A}(ffp_n,gp,a)=1$ and $\lim_{n\to\infty}\mathbb{A}(ggp_n,fp,a)=1$,

Where $\{p_n\}$ W that gives $\lim_{n\to\infty}fp_n=\lim_{n\to\infty}gp_n=p$ and $p\in W$ where $a>0$.

Lemma 11. *[19] Let $(W,\mathbb{A},*)$ be a fuzzy b-metric space and for a given $s\geq 1$, if there exists $k\in\left(0,\dfrac{1}{s}\right)$ such that $\mathbb{A}(p,q,ka)\geq\mathbb{A}(p,q,a)$ for all $p,q\in W$ and $a>0$ then $p=q$.*

□

Lemma 12. *[20]If $\{q_n\}$ be a sequence in a fuzzy b-metric space $(W,\mathbb{A},*)$ with $s\geq 1$ where $k\in\left(0,\dfrac{1}{s}\right)$ such that $\mathbb{A}(q_{n+2},q_{n+1},ka)\geq\mathbb{A}(q_{n+1},q_n,a)$ for $a>0$ and $n=1,2,3,...$, then $\{q_n\}$ is a Cauchy sequence in W.*

3. Main Result

Theorem 13. *Suppose $(W,\mathbb{A},*)$ be a complete fuzzy b-metric space with $a*a\geq a$, for all $a\in[0,1]$ and there exists $s\geq 1$ such that $k\in(0,\frac{1}{s})$. Assume that $f,g,h,j:W\to W$ be the mappings with the following properties:*

(i) $f(W)\subseteq j(W), g(W)\subseteq h(W)$,

(ii)

$$\mathbb{A}(fp,gq,ka)\geq\mathbb{A}(hp,jq,a)*\mathbb{A}(fp,hp,a)*\mathbb{A}(gq,jq,a)*\mathbb{A}(fp,jq,a) \tag{1}$$

for all $p,q\in W$ and $a>0$,

(iii) h and j are continuous.

If (f,h) and (g,j) possess the limit point as typical within the range of h and are (K) compatible, then all the mappings defined as above have a unique common fixed point.

Proof. Let us choose for any $p_0\in W$. As $f(W)\subseteq j(W)$ also $g(W)\subseteq h(W)$, there exist $p_1,p_2\in W$ such that $fp_0=jp_1$ and $gp_1=hp_2$. If we define a sequence inductively, as $\{p_n\}$ and $\{q_n\}$ in W for which

$$q_{2n+1}=fp_{2n}=jp_{2n+1},$$
$$q_{2n+2}=gp_{2n+1}=hp_{2n+2},$$
$$\text{for all } n=0,1,2,\ldots$$

Now from 1, by putting $p=p_{2n}, q=p_{2n+1}$ we get

$$\mathbb{A}(fp_{2n},gp_{2n+1},ka)\geq\{\mathbb{A}(hp_{2n},jp_{2n+1},a)*\mathbb{A}(fp_{2n},hp_{2n},a)*\mathbb{A}(gp_{2n+1},jp_{2n+1},a)*\mathbb{A}(fp_{2n},jp_{2n+1},a)\}$$

Which gives

$$\mathbb{A}(q_{2n+1}, q_{2n+2}, ka) \geq \{\mathbb{A}(q_{2n}, q_{2n+1}, a) \\ *\mathbb{A}(q_{2n+1}, q_{2n}, a) * \mathbb{A}(q_{2n+2}, q_{2n+1}, a) \\ *\mathbb{A}(q_{2n+1}, q_{2n+1}, a)\}$$

This gives

$$\mathbb{A}(q_{2n+1}, q_{2n+2}, ka) \geq \mathbb{A}(q_{2n}, q_{2n+1}, a) * \mathbb{A}(q_{2n+2}, q_{2n+1}, a)$$

From lemma 11, we have,

$$\mathbb{A}(q_{2n+1}, q_{2n+2}, ka) \geq \mathbb{A}(q_{2n}, q_{2n+1}, a)$$

Similarly we have,

$$\mathbb{A}(q_{2n+2}, q_{2n+3}, ka) \geq \mathbb{A}(q_{2n+1}, q_{2n+2}, a)$$

Then we can say

$$\mathbb{A}(q_{n+1}, q_{n+2}, ka) \geq \mathbb{A}(q_n, q_{n+1}, a)$$

Now by repeating above inequality, we get

$$\mathbb{A}(q_{n+1}, q_{n+2}, a) \geq \mathbb{A}\left(q_n, q_{n+1}, \frac{a}{k}\right) \\ *\mathbb{A}\left(q_{n-1}, q_n, \frac{a}{k^2}\right) * \ldots * \mathbb{A}\left(q_1, q_2, \frac{a}{k^n}\right)$$

But

$$\mathbb{A}\left(q_1, q_2, \frac{a}{k^n}\right) \to 1 \quad \text{as } n \to \infty.$$

Hence,

$$\mathbb{A}(q_{n+1}, q_{n+2}, ka) \geq \mathbb{A}(q_n, q_{n+1}, a) \tag{2}$$

By Lemma 12 $\{q_n\}$ is a Cauchy sequence in W.

Since $(W, \mathbb{A}, *)$ is complete, so the sequence $\{q_n\}$ converges to some point $w \in W$. Also its sub-sequences $\{fp_{2n}\}$, $\{hp_{2n+2}\}$, $\{jp_{2n+1}\}$, and $\{gp_{2n+1}\}$ converge to w. Since the pairs (f, h) and (g, j) share the common limit in the range h then there exist two sequences $\{p_n\}$ and $\{q_n\}$ in W such that

$$\lim_{n\to\infty} fp_n = \lim_{n\to\infty} hp_n = \lim_{n\to\infty} gq_n = \lim_{n\to\infty} jq_n = hw \tag{3}$$

Since $\{p_n\}$ and $\{q_n\}$ converge to $w \in W$, and using the continuity of h and j, we have:

$$\lim_{n\to\infty} hp_n = hw$$
$$\lim_{n\to\infty} jq_n = jw$$

Now at first we will show that $fw = hw$

putting $p = w$ and $q = q_n$ in 1, we have

$\mathbb{A}(fw, gq_n, ka) \geq \{\mathbb{A}(hw, jq_n, a) * \mathbb{A}(fw, hw, a) * \mathbb{A}(gq_n, jq_n, a) * \mathbb{A}(fw, jq_n, a)\}$

taking limit as$n \to \infty$, we have

$\mathbb{A}(fw, hw, ka) \geq \{\mathbb{A}(hw, hw, a) * \mathbb{A}(fw, hw, a) * \mathbb{A}(hw, hw, a) * \mathbb{A}(fw, hw, a)\}$

So,

$$\mathbb{A}(fw, hw, ka) \geq \mathbb{A}(fw, hw, a)$$

Then by lemma 11,

$$fw = hw \tag{4}$$

Since $f(W) \subset j(W)$ then there exists $v \in W$ such that $fw = jv$.

from 1 by putting $p = w$ and $q = v$, we get

$$\mathbb{A}(fw, gv, ka) \geq \{\mathbb{A}(hw, jv, a) \\ *\mathbb{A}(fw, hw, a) * \mathbb{A}(gv, jv, a) * \mathbb{A}(fw, jv, a)\} \tag{5}$$

Now from 4 and 5 we get,

$\mathbb{A}(jv, gv, ka) \geq \{\mathbb{A}(jv, jv, a) * \mathbb{A}(jv, jv, a) * \mathbb{A}(jv, gv, a) * \mathbb{A}(jv, jv, a)\}$

Hence $\mathbb{A}(jv, gv, ka) \geq \mathbb{A}(jv, gv, a) * 1 * 1 * 1$ which implies $\mathbb{A}(jv, gv, ka) \geq \mathbb{A}(jv, gv, a)$

from the lemma 11, we have

$$jv = gv \tag{6}$$

From above 4,5 and 6 we have,

$$w = hw = jv = gv \tag{7}$$

Now we will prove that $fw = w$

Putting $p = w$ and $q = p_{2n+1}$, in 1

$$\mathbb{A}(fw, gp_{2n+1}, ka) \geq \mathbb{A}(hw, jp_{2n+1}, a) \\ *\mathbb{A}(fw, hw, a) * \mathbb{A}(gp_{2n+1}, jp_{2n+1}, a) \\ *\mathbb{A}(fw, jp_{2n+1}, a)$$

Taking limit as $n \to \infty$ we get,

$$\mathbb{A}(fw, w, ka) \geq \mathbb{A}(hw, w, a) * \mathbb{A}(fw, hw, a) \\ *\mathbb{A}(w, w, a) * \mathbb{A}(fw, w, a) \\ \mathbb{A}(fw, w, ka) \geq \mathbb{A}(fw, w, a)$$

By Lemma 11, $fw = w$,

from 7 we get, $w = jv = gv$.

Now since (f, h) and (g, j) are compatible of type (K), then

$$\lim_{n\to\infty} ffp_{2n} = hw, \lim_{n\to\infty} hh(p_{2n+2}) = fw$$

$$\lim_{n\to\infty} gg(p_{2n+1}) = jw \quad \text{and} \quad \lim_{n\to\infty} jj(p_{2n+1}) = gw$$

Now from 1 we have,

$$\mathbb{A}(ffp_{2n}, ggp_{2n+1}, ka) \geq \mathbb{A}(hfp_{2n}, jgp_{2n+1}, a) \\ *\mathbb{A}(ffp_{2n}, hfp_{2n}, a) * \mathbb{A}(ggp_{2n+1}, jgp_{2n+1}, a) \\ *\mathbb{A}(ffp_{2n}, jg_{2n+1}, a)$$

Taking limit as $n \to \infty$ and using 3, we get

$$\mathbb{A}(hw, jw, ka) \geq \mathbb{A}(jw, hw, a) * \mathbb{A}(hw, hw, a) \\ *\mathbb{A}(jw, jw, a) * \mathbb{A}(hw, jw, a)$$

which gives,

$\mathbb{A}(hw, jw, ka) \geq \mathbb{A}(jw, hw, a) * 1 * 1 * \mathbb{A}(jw, hw, a)$

Then we have $\mathbb{A}(hw, jw, ka) \geq \mathbb{A}(jw, hw, a) * \mathbb{A}(jw, hw, a)$

Hence,

$\mathbb{A}(hw, jw, ka) \geq \mathbb{A}(jw, hw, a)$

Therefore we have, $jw = hw$.

Combining all the above results, we get $w = fw = gw = hw = jw$.

From this we conclude that w is a fixed point of f, g, h and j as common.

Now we will show that there is unique fixed point:

Let w_1 be another common fixed point of f, g, h, and j. Then we have

$$w_1 = fw_1 = gw_1 = hw_1 = jw_1$$

Then by inequality 1, putting $p = w$ and $q = w_1$, we get

$$\begin{aligned} \mathbb{A}(fw, gw_1, ka) \geq \mathbb{A}(hw, jw_1, a) * \mathbb{A}(fw, hw, a)* \\ \mathbb{A}(gw_1, jw_1, a) * \mathbb{A}(fw, jw_1, a) \\ \mathbb{A}(w, w_1, ka) \geq \mathbb{A}(w, w_1, a) \\ \implies w = w_1 \end{aligned}$$

Thus w is the fixed point as required for the given mappings. □

Above theorem can be verified from the following example where the continuous t-norm is defined in terms of the product.

Example 14. Let us define the functions $f, g, h,$ *and* $j : W \to W$ as below where $W = [0, 10]$

$$f(p) = \begin{cases} 4, & p \leq 5, \\ 5, & p > 5, \end{cases}$$

$$g(p) = \begin{cases} 4, & p \leq 7, \\ 5, & p > 7, \end{cases}$$

$h(p) = p, j(p) = p$ and a fuzzy b-metric space is defined as:

$\mathbb{A}(p, q, a) = e^{-\dfrac{|p-q|}{a^2}}$

When $p = 2, \quad q = 3, \quad a = 1, \quad k = 0.5, \quad s = 2, \quad f(2) = 4, \quad g(3) = 4, \quad g(2) = 2, \quad g(3) = 3, \quad j(2) = 2, \quad j(3) = 3, \quad h(2) = 2, \quad h(3) = 3$, then LHS of the inequality 1 i.e.

$$\begin{aligned} \mathbb{A}(f(p), g(q), ak) \geq \mathbb{A}(hp, jq, a) * \mathbb{A}(fp, hp, a) \\ *\mathbb{A}(gq, jq, a) * A(fp, jq, a) \end{aligned}$$

becomes

$\mathbb{A}(Fp, gq, ka) = \mathbb{A}(f(2), g(3), 1 \times 0.5) = \mathbb{A}(4, 4, 0.5) = 1$

$$\text{RHS} = \mathbb{A}(hp, jq, a) * \mathbb{A}(fp, hp, a) * \mathbb{A}(gq, jq, a) * \mathbb{A}(fp, jq, a)$$

$$= \mathbb{A}(h(2), j(2), 1) * \mathbb{A}(f(2), h(2), 1)$$

$$*\mathbb{A}(g(3), j(2), 1) * \mathbb{A}(f(2), j(3), 1)$$

$$\mathbb{A}(2, 2, 1) * \mathbb{A}(4, 2, 1) * \mathbb{A}(4, 2, 1) * \mathbb{A}(4, 3, 1)$$

$$1 \cdot e^{-2} \cdot e^{-2} \cdot e^{-1} = 0.000651$$

Thus $1 \geq 0.000651$. Hence the inequality is satisfied.

Now to test the (K) compability define a sequence by $p_n = 4 + \dfrac{1}{n}$

Then

$$\lim_{n\to\infty} f(p_n) = \lim_{n\to\infty} f\left(4 + \frac{1}{n}\right) = f(4) = 4$$

and

$$\lim_{n\to\infty} h(p_n) = \lim_{n\to\infty} \left(4 + \frac{1}{n}\right) = h(4) = 4$$

Therefore

$$\lim_{n\to\infty} f(p_n) = \lim_{n\to\infty} h(p_n) = 4 \in W$$

Now,

$$\lim_{n\to\infty} \mathbb{A}(ffp_n, h(p), a) = \lim_{n\to\infty} \mathbb{A}(4, 4, a) = 1$$

Also

$$\lim_{n\to\infty} \mathbb{A}(hhp_n, h(p), a) = \lim_{n\to\infty} \mathbb{A}(4, 4, a) = 1.$$

Hence the pair (f, h) is (K) compatible. In similar way we can say that the pair (g, j) is also (K) compatible. By definition, the functions h and j are continuous.

Where 4 is a common fixed point.

From the above discussion we can state the following corollary.

Corollary 15. *Let us consider a complete fuzzy b-metric space* $(W, \mathbb{A}, *)$ *with the self mappings* $f, g, h,$ *and* j *and for a given real number* $s \geq 1$ *having the following properties*

(a) $f(W) \subseteq j(W)$ and $g(W) \subseteq h(W)$

(b) There exists $k \in \left(0, \dfrac{1}{s}\right)$ and for every $u, v \in U$ and $a > 0$,

$$\begin{aligned} \mathbb{A}(fp, gq, ka) \geq \min\{\mathbb{A}(hp, jq, a), \mathbb{A}(fp, hp, a), \\ \mathbb{A}(gq, jq, a), \mathbb{A}(fp, gq, a), \mathbb{A}(hp, gq, a)\} \end{aligned}$$

(c) The mappings g and j are weakly compatible if (f, h) is (K) compatible. If any one of the given mappings is continuous, consequently all these mappings have a unique common fixed point in W.

4. Conclusion

Using the notions of fuzzy b-metric space and their basic terminologies, we proved a common fixed point theorem in complete fuzzy b-metric space applying compatible mapping of type (K) introduced by K.B. Manandhar, K. Jha and G. Porru [15] in 2014. Due to the flexible triangle inequality and the resulting topological issues, the result is more interesting than the previous results in fuzzy metric spaces. Our result improves and extends the results of K.B. Manandhar, K. Jha and G. Porru [15], R. Umamaheshwar and B. Vijayabasker [21], K. Jain and J. Kaur [22], and other similar results in literature.

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