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A steady MHD thin film flow of carreau fluid down an inclined plane with viscous and magnetic dissipation under slip boundary

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Abstract

This paper presents an investigation of a steady magnetohydrodynamic (MHD) thin-film flow of a Carreau fluid down an inclined plane with viscous and magnetic dissipation under slip boundary condition. The derived non-linear ODEs that govern the flow of both the velocity and temperature profile were solved using perturbation method with MAPLE software. The impact of few parameters like Magnetic field (M), Brinkmann number (Br), Gravitational force (G), and the slip parameter (β), on the velocity and temperature profiles were identified and demonstrated graphically. The result shows that the velocity and temperature reduced significantly when the Magnetic field parameter increases and both increases as the slip parameter β increases. As there is rise in the gravitational force G, the velocity diminishes while the temperature intensifies. There was an increase in velocity and temperature circulation of the moving fluid due to the rise in the slip parameter, signifying enhanced heat transfer by upward flow of the fluid. Increase in Brinkmann has no effect on the velocity of the fluid.

Keywords: Magnetohydrodynamic (MHD); Carreau fluid; Non-Newtonian fluids; Viscous and magnetic dissipation; Maple software; Slip boundary condition.

1. Introduction

Fluids can be characterized into Newtonian and Non-Newtonian. Newtonian fluids comply with Newton's law of viscosity. Air and water are Newtonian fluids. Isaac Newton's equation of viscosity illustrates the linear connection between the shear strain rate and shear stress of a particular fluid. A Newtonian fluid has a non-varying viscosity, zero shear rate and zero shear stress, that is, its viscosity remains constant irrespective of the shear rate.

Non-Newtonian fluids such as Ketchup, paint, glue, tar, blood etc. have their viscosity increased or decreased when shear stress is applied. Dilatant fluid as one of the categories of non-Newtonian fluid can be referred to as shear thickening since its viscosity increases as the shear rate increases while that of pseudoplastic fluid decreases. Carreau fluids are generalized non-Newtonian fluids in which its thickness depend upon the shear rate.

Through Magnetic dissipation, the work done by the moving thin-film fluid is transformed into thermal energy and with slip condition, the flowing fluid particles move with respect to the surface due to the effect of viscosity.

The impacts of mass transfer on Magnetohydrodynamic oscillatory movement of Carreau fluid over a slanted permeable channel affected by temperature was studied by [1], the effects of some parameters on the fluid movement were analyzed and obtained using the perturbation method. An amplified fluid flow due to increased energy and the generated heat and fluid movement reduced by increase in chemical reaction parameters was observed.

The flowing of a Carreau fluid over a wall-driven angle considering Taylor's classical paint scrapping problem as a framework with velocity u was investigated by [2]. It was discovered that the shear rate is proportional to the detachment from the line of zero shear and also for less than 2.2 rad corner angles and also the effects of Carreau fluid in the far-field are significant.

The effects of non-constant viscosity on the flow of a responsive fluid over a convective medium were studied by [3]. A series solution of modified decomposition method was used to solve the governing equation. The resulting graphs shows that with the increase in the Frank-Kamenettski parameter, viscous heating parameter, heat source parameter and the fluid temperature increase. Improvement in the motion of the fluid due to rise in the viscosity and a decrease in the fluid motion with increase in thermal radiation parameter values and convective cooling was also noticed.

A comparison resolution of time-dependent Magnetohydrodynamic fluid film movement over an extending sheet with non-constant physical properties was studied by [4]. The resulting non-linear equation was solved analytically and numerically using homotopy and shooting methods. Effects of some physical parameters such as variable viscosity, Hartmann number, thermal conductivity, film thickness, and Prandtl parameter on the temperature and velocity profiles were studied and the outcome shows that as viscosity rises, the flow velocity reduces while the temperature increases. Also, Hartmann number rises in value, the flow velocity increases with no effect on the fluid temperature.

The existence of unlimited shear rate viscidness in the flow of a steady two-dimensional Carreau fluid over a moving/stationary wedge was investigated by [5]. The non-linear partial differential equations (PDE) are solved numerically with Runge-Kutta method. It was observed that there is rise in the fluid velocity and there was reduction in the temperature field as the wedge angle parameter increases.

Related to this research work, a steady MHD thin film movement

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of Carreau fluid down a slipped-down smooth plane with viscous and magnetic dissipation under slip boundary was carried out by [6]. The derived equations were solved analytically using MAPLE software and the effect of some physical parameters on the fluid velocity and temperature were examined. The result indicates that as the Magnetic field parameter M increases the velocity and temperature reduces and both increases as the inclination angle parameter increases. As there is rise in the gravitational force, the velocity decreases while the temperature increases.

The effect of heat generation and mass transfer on the flow of Carreau fluid over a porous stretching medium with slip boundary condition considering some physical parameters such as applied magnetic field, thermal radiation, cross diffusion and suction/injection effects was investigated by [7]. The resulting equations were solved numerically. It was discovered that Dufour and Soret parameters regulate the generated heat and the mass transfer rate. Also, the thermal boundary layer thickness was enhanced by non-linear thermal radiation.

The consequence of heat transfers on Magnetohydrodynamics oscillatory flow for Carreau-Yasuda fluid through a permeable channel was examined [8]. Poiseuille flow and Couette flow geometries were involved and solved by using the perturbation method. It was observed that the effect of some identified constraints, such as Weissenberg number, Darcy number, Reynold number, Peclet number, Magnetic parameter, radiation parameter for the velocity and recurrence of the oscillation are on the fluid movement illustrated using MATHEMATICA software.

The presence of PHF and PCF in the flow of a three-dimensional strained viscid fluid was studied by [9]. Mathematical model was derived with the effects of viscous dissipation, chemical reaction, and Joule heating. The impacts of some parameters on velocity distributions and temperature of the fluid were shown graphically. An increase in magnetic parameter, increases the temperature and the concentration fields. Increase in the values of ratio parameter reduces the temperature and the boundary layer thicknesses. Prandtl number is a decreasing function and Eckert number is an increasing function of the temperature.

Magneto Casson-Carreau fluid movement over a circular permeable tube with partial slip was studied by [10], a comparative analysis of two-dimensional heat transfer was considered and model for the two fluids were formulated. The ordinary differential equations obtained were solved by KBM. It was observed from the result that there is decrease in the velocity of both the Carreau and Casson fluid due to increase in magnetic number and Casson fluid has higher qualities contrasted with Carreau fluid in disparity of magnetic number.

The use of the general dispersion model in the dispersion of unsteady solute in Non-Newtonian fluid movement in a pipe in the presence and absence of wall absorption and reaction at the wall of the tube was investigated by [11]. Different non-Newtonian fluid mockups were used. It was discovered that both the solute dispersion and flow velocity increases and also decreases with solute concentration as Weissenberg number 'We' rises. Regular and homotopy perturbation method were used to solve the subsequent ODE. The result shows that both methods have the same outcomes and increase in magnetic field reduces the velocity distribution and temperature circulation increases while as angle of inclination increases, both the velocity and temperature distribution of the fluid increases and also slip parameter is an increasing function of both velocity and temperature distribution.

The flow of different electrically and magnetic induced MHD fluids have been worked upon by many researchers. The progression of incompressible Carreau liquid between equal plates with slip conditions [12]. The flow geometry was represented in a coordinate system, and the non-dimensional problem was solved using analytical method. An increase in magnetic parameter with velocity decrease function of radiation parameter and brinkman number was observed. It was also found that there was increment in the velocity and temperature distribution of the moving fluid due to rise in the inclination of the plate, indicating enhanced heat transfer by ascendant flow of the fluid. Dual solutions for the movement of a non-Newtonian MHD Carreau fluid over the shrinking surface using mathematical modeling of energy and mass transfer were analyzed [13]. Dual solutions were obtained for all the parameters used and it was observed that both the concentration and the temperature field showed same effect on both the solutions for temperature ratio and the velocity ratio parameters.

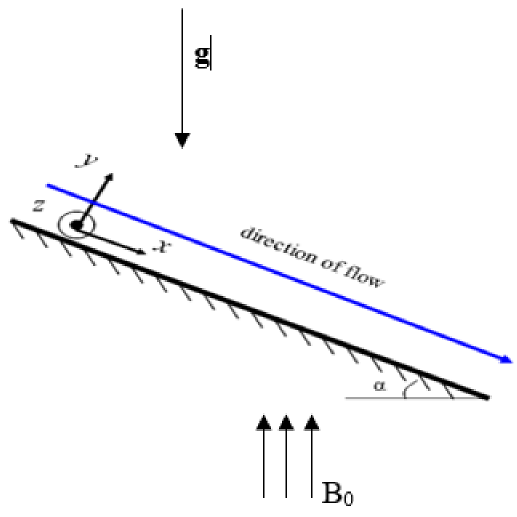

Figure 1: A systematic diagram of the physical model.

In this work, we investigate the slip effect of MHD Carreau fluid flow down an inclined surface with viscous and magnetic dissipation. Perturbation method was used to solve the non-linear ODE that governs the flow. The influence of each of the non-dimensionless parameters on the momentum and the temperature of the fluid and the convergence of the solution were shown graphically.

Consider the MHD flow of a Carreau fluid with a uniform viscosity down a surface that slopes at an angle α with slip condition. The constitutive equations are as follows:

$$s = \mu(\dot{\gamma})\dot{\gamma} = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\Gamma\dot{\gamma})^a]^{(n-1)/2} \quad (1)$$

where 'a' is the Carreau parameter when it is equal to 2.
s -shear stress
$\dot{\gamma}$ - shear rate respectively
μ_0 - zero-shear rate viscosity
μ_∞-infinite shear rate viscosity
Γ - material time constant
n - power-law exponent region

The constitutive governing equations for the momentum and energy are:

$$\frac{\partial S}{\partial y} - \sigma B_0^2 u + \rho g sin\alpha = 0 \quad (2)$$

with boundary conditions:

$$u = \beta s \text{ at } y = 0, \ \frac{du}{dy} = 0 \text{ at } y = h \quad (3)$$

and

$$k\frac{(\partial^2 T)}{(\partial y^2)} + s\frac{\partial u}{\partial y} + \sigma B_0^2 u^2 = 0 \quad (4)$$

with boundary conditions:

$$T = T_0 \text{ at } y = 0, T = T_1 \text{ at } y = h \quad (5)$$

where β is the slip coefficient and s is the total stress tensor, k and σ are the thermal and electrical conductivity of the fluid.

Substituting equation (1) into equation (2) - (5) and introduce the following non-dimensionless

quantities $\bar{y} = \frac{y}{h}, \bar{x} = \frac{x}{h}, \bar{u} = \frac{uh}{\mu_0}, u = \frac{(\bar{u}\mu_0)}{h}, p = \frac{(\bar{p}h)}{\mu_0}, \bar{p} = \frac{(p\mu_0)}{h}, T = \frac{(\theta-\theta_0)}{(\theta_0-\theta_1)}$ to yield

$$\frac{\partial^2 u}{\partial y^2} + \frac{3}{2}We(n-1)\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y} - Mu + G = 0 \quad (6)$$

with boundary conditions

$$u = \beta\left[\frac{\partial u}{\partial y} + \frac{1}{2}We(n-1)\left(\frac{\partial u}{\partial y}\right)^3\right]$$
$$\text{at } y = 0 \text{ and } \frac{\partial u}{\partial y} = 0 \text{ at } y = 1 \quad (7)$$

$$\frac{\partial^2 \theta}{\partial y^2} + Br\left[\left(\frac{\partial u}{\partial y}\right)^2 + We\left(\frac{\partial u}{\partial y}\right)^4 + Mu^2\right] = 0 \quad (8)$$

with boundary conditions

$$\theta = 0 \text{ at } y = 0 \text{ and } \theta = 1 \text{ at } y = 1 \quad (9)$$

where $M = \frac{\sigma B_0^2 h}{\mu_0}$ is the Magnetic parameter, $G = \frac{\rho g \sin \alpha\ h}{\mu_0}$ is the Gravitational parameter $We = \frac{\lambda^2(1-\sigma)}{h^3}$ is the Weissenberg number and$Br = (\mu_0^3)\frac{\mu_0^3}{h^2 K(\theta_1-\theta_0)}$ is the Brinkman number.

2. Method

The numerical simulation was conducted using MAPLE 20. The derived nonlinear PDEs that govern the flow of the fluid and the heat transfer was solved using perturbation method by expanding the solution in powers of a small parameter ε for both the momentum and the temperature which are as follows:

Zero-order equation for the momentum and temperature

$$\frac{\partial^2 u_0}{\partial y^2} - Mu_0 + G = 0 \quad (10)$$

with boundary conditions:

$$u_0 - \beta\frac{\partial u_0}{\partial y} = 0 \text{ at } y = 0 \text{ and } \frac{\partial u_0}{\partial y} = 0 \text{ at } y = 1 \quad (11)$$

And this;

$$\frac{\partial^2 \theta_0}{\partial y^2} + Br\left[\left(\frac{\partial u_0}{\partial y}\right)^2 + Mu_0^2\right] = 0 \quad (12)$$

with boundary conditions:

$$\theta_0 = 0 \text{ at } y = 0 \text{ and } \theta_0 = 1 \text{ at } y = 1 \quad (13)$$

First-order equation for the momentum and temperature

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{3}{2}(n-1)\left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_0}{\partial y^2} - Mu_1 = 0 \quad (14)$$

with boundary conditions

$$u_1 - \beta\frac{\partial u_1}{\partial y} - \frac{1}{2}\beta(n-1)\left(\frac{\partial u_0}{\partial y}\right)^3 = 0$$
$$\text{at } y = 0 \text{ and } \frac{\partial u_1}{\partial y} = 0 \text{ at } y = 1 \quad (15)$$

And this;

$$\frac{\partial^2 \theta_1}{\partial y^2} + Br\left[\left(\frac{\partial u_0}{\partial y}\right)^4 + 2\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y} + 2u_0u_1\right] = 0 \quad (16)$$

with boundary conditions

$$\theta_1 = 0 \text{ at } y = 0 \text{ and } \theta_1 = 0 \text{ at } y = 1 \quad (17)$$

Using finite difference method, the PDEs were reduced to ODEs which were then solved numerically with the boundary conditions using MAPLE commands.

Solving equations (10) and (12) with boundary conditions (12) and (14) and equation (16) and (18) with boundary conditions (17) and (19), the following momentum and temperature equations for both the zeroth and first order were obtained:

Momentum equations

$$u_0 = e^{\sqrt{M}y}c_{10} + e^{-\sqrt{M}y}c_9 + \frac{G}{M} - \beta(\sqrt{M}e^{\sqrt{M}y}c_{10} - \sqrt{M}e^{-\sqrt{M}y}c_9) \quad (18)$$

$$u_1 = a_{25}\left(e^{\sqrt{M}y}\right)^3 + \left(a_{26} + c_{12} - \beta\sqrt{M}c_{12} + a_{27}y\right)e^{\sqrt{M}y} + \frac{a_{28}y + a_{29} + \beta\sqrt{M}c_{11} + c_{11}}{e^{\sqrt{M}y}} + \frac{a_{30}}{\left(e^{\sqrt{M}y}\right)^3} \quad (19)$$

Temperature equations

$$\theta_0 = -\frac{1}{4}\frac{a_{31}e^{2\sqrt{M}y}}{M} - \frac{a_{32}e^{\sqrt{M}y}}{M} - \frac{a_{34}e^{-\sqrt{M}y}}{M} - \frac{1}{4}\frac{a_{35}e^{-2\sqrt{M}y}}{M} - \frac{1}{2}a_{33}y^2 + c_{13}y \quad (20)$$

$$\begin{aligned}\theta_1 \quad = \quad & -\frac{1}{16}\frac{a_{36}e^{4\sqrt{M}y}}{M} - \frac{1}{4}\frac{a_{38}e^{2\sqrt{M}y}y}{M} + \frac{1}{4}\frac{a_{38}e^{2\sqrt{M}y}}{M^{3/2}} - \frac{1}{16}\frac{a_{37}e^{2\sqrt{M}y}}{M} - \frac{a_{40}e^{-\sqrt{M}y}y}{M} - \frac{a_{39}e^{-\sqrt{M}y}}{M} - \\ & \frac{1}{4}\frac{a_{42}e^{-2\sqrt{M}y}y}{M} - \frac{1}{4}\frac{a_{42}e^{-2\sqrt{M}y}}{M^{3/2}} - \frac{1}{4}\frac{a_{41}e^{-2\sqrt{M}y}y}{M} - \frac{1}{16}\frac{a_{43}e^{-4\sqrt{M}y}}{M} - \frac{2}{9}\frac{BrGa_{23}e^{-3\sqrt{M}y}}{M} - \\ & \frac{2}{9}\frac{BrGa_{21}e^{3\sqrt{M}y}}{M} - \frac{a_{46}e^{\sqrt{M}y}y}{M} + \frac{2a_{46}e^{\sqrt{M}y}}{M^{3/2}} - \frac{a_{45}e^{\sqrt{M}y}}{M} - \frac{1}{2}a_{44}y^2 + c_{15}y + c_{16}\end{aligned} \quad (21)$$

where a_1,a_2,........,a_{46} andc_1,c_2,........,$c_1$6 are constant.

These equations were substituted with values to obtain both the numerical and analytical results.

3. Results

Table 1 and 2 shows the exact and numerical solutions derived for both the velocity (u) and the temperature (θ) profiles for values of the Magnetic field parameter M = 2 and M=4 when other parameters remain constant. Values for other parameters such as the Gravitational force G, Brinkmann number Br and the slip parameter were also derived for both the exact analytical result and numerical solution. Table 2 shows the values for the velocity and temperature of the moving fluid when affected by the physical parameters and there is an increase in the slip parameter β from 0.5 to 1.0

Table 1 and 2 illustrates the validity of the results obtained from perturbation method and numerical method in MAPLE 20.

Comparing equations (18) - (21) of this present work to equations (27) - (31) in [6], It was observed that by making the slip parameter β=0 the same results were obtained for both slip and no slip boundary conditions for and , which shows the validity of the whole test and analysis.

Fig. 2 illustrates the influence of magnetic parameter M on the velocity and temperature distribution of the fluid when G = 2, n = 2, Br = 1 and ε=0.001. It was noticed from the graphs that both the velocity and temperature decrease as M increases. The effects of increasing values of M are to reduce the fluid velocity and also reduce the thickness of the boundary layer. Then, with rise in the magnetic field parameter, the rate of transportation will be reduced as the fluid flows down an inclined plane.

Fig. 3 illustrates the graphical illustration of both the velocity u and temperature θ distribution for different values of G. It was observed from the graph that as the gravitational parameter G increases, the temperature distribution intensifies and the velocity declines.

Fig. 4 illustrates that the temperature distribution of the flowing fluid rises as the Brinkman number intensifies while the Brinkmann number does not have effect on the velocity distribution of the fluid.

Fig. 5 and Fig. 6 shows that as there is rise in the slip parameter β, both the velocity and temperature of the moving fluid increases.

4. Conclusion

In this study, the MHD flow of a thin film fluid with viscous and magnetic dissipation down an inclined surface with slip boundary conditions were examined and analyzed. Governing equations were derived for both momentum and temperature. The influence of some physical dimensionless parameters such as Magnetic field (M), Gravitational force (G), and Brinkmann number (Br), and slip parameter (β) on both the velocity and temperature distribution were observed, computed and represented graphically. The results derived from analytical method were validated numerically using finite difference method. It was also discovered that:

-The temperature of the moving fluid drastically increases till it converges at 1.0000 for both values of M=2 and M=4 and for the slip parameter β from 0 to 0.5

-The velocity profile decreases as the magnetic parameter M and the slip parameter βincreases with a maximum decrease of 0.141%

-The temperature distribution of the fluid increases when the slip parameter increases with a maximum increase of 0.25%

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Table 1: Exact solution and numerical results in MAPLE 20 for M=2 and M=4 when G = 2, n = 2, Br = 1, β= 0.5 and ε=0.001.

	M=2				M=4			
	Exact solution		Numerical results using MAPLE 20		Exact solution		Numerical results using MAPLE 20	
y	u	θ	u	θ	u	θ	u	θ
0	0.5702	0	0.5702	0	0.3394	0	0.3394	0
0.2	0.6622	0.2916	0.6622	0.2916	0.3900	0.2594	0.3900	0.2594
0.4	0.7271	0.5420	0.7270	0.5420	0.4227	0.4929	0.4227	0.4929
0.6	0.7701	0.7472	0.7720	0.7472	0.4429	0.6973	0.4420	0.6973
0.8	0.7946	0.9039	0.7946	0.9039	0.4538	0.8701	0.4538	0.8702
1	0.8026	1.0099	0.8026	1.0000	0.4573	1.0000	0.4573	1.0000

Table 2: Test for analytical and numerical results in MAPLE 20 for M=2 and M=4 when G = 2, n = 2, Br = 1, ε=0.001, and β= 0.

	M=2				M=4			
	Exact solution		Numerical results		Exact solution		Numerical results	
y	$u(y)$	$\theta(y)$	$u(y)$	$\theta(y)$	$u(y)$	$\theta(y)$	$u(y)$	$\theta(y)$
0	0	0	0	0	0	0	0	0
0.2	0.2134	0.2601	0.2142	0.2600	0.1571	0.2390	0.1574	0.2390
0.4	0.3643	0.4826	0.3653	0.4826	0.2590	0.4570	0.2593	0.4570
0.6	0.4644	0.6784	0.4653	0.6785	0.3219	0.6574	0.3222	0.6575
0.8	0.5214	0.8508	0.5223	0.8508	0.3561	0.8390	0.3563	0.8390
1	0.5399	0.9999	0.5408	1.0000	0.3669	1.0000	0.3670	1.0000

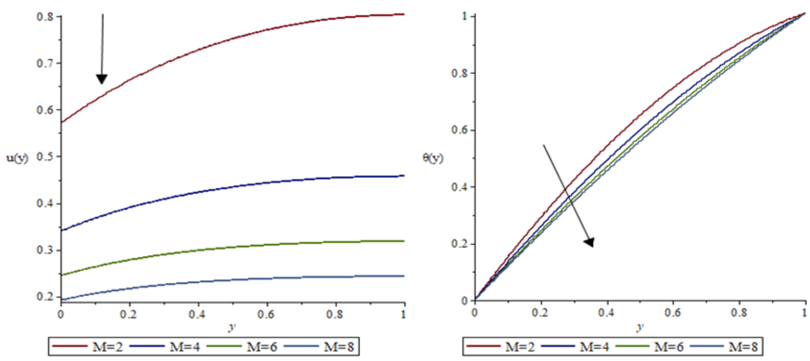

Figure 2: Effects of magnetic parameter on the velocity profile (a) and temperature profile (b).

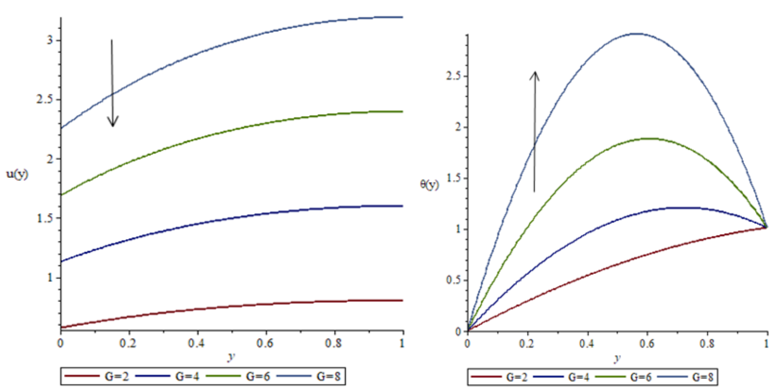

Figure 3: Effects of gravitational parameter on the velocity profile (a) and temperature profile (b).

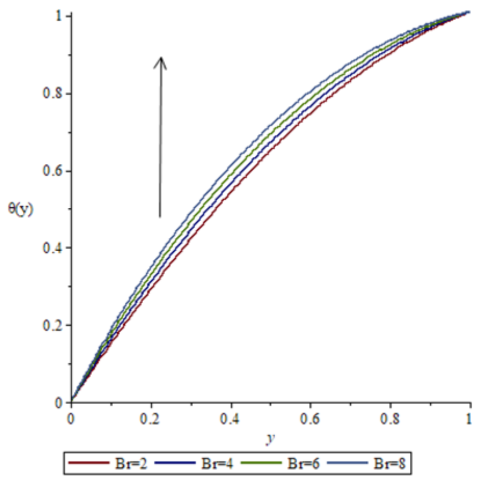

Figure 4: Effects of Brinkmann number on the temperature profile.

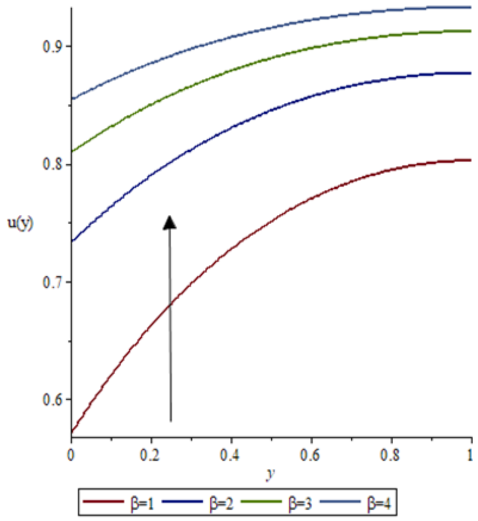

Figure 5: Effects of slip parameter on the velocity profile.

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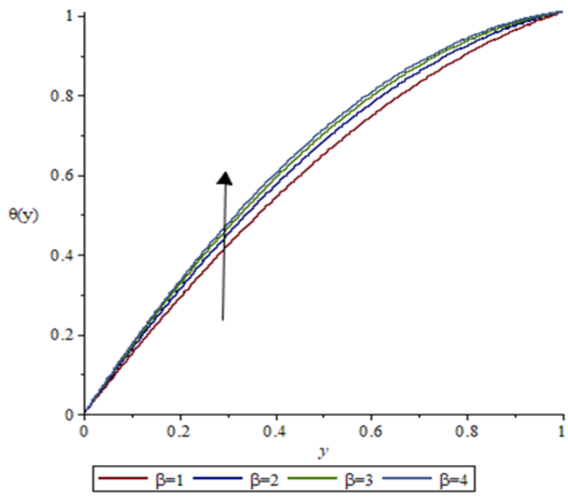

Figure 6: Effects of slip parameter on the temperature profile.

Table 3: List of symbols and their meaning.

Symbols	Meaning	Units
S	Shear stress: This occurs when a force is applied to a material.	N/m^2
$\dot{\gamma}$	Shear rate: The rate at which the fluid is deformed by shear stress.	s^{-1}
μ_0	Zero shear rate: A fluid is at rest I.e no deformation of the fluid.	$0s^{-1}$
μ_∞	Infinite shear rate: This occurs when the fluid deformed at an extremely high rate.	s^{-1}
Γ	Material time constant: It measures the time taken for the fluid to relax or respond to external forces.	S
n	Power law exponent: This describes the relationship between shear stress and shear rate in non-Newtonian fluid.	None, because it is dimensionless
B_0	Magnetic force: A force that interact between magnetic field and magnetic moment.	Newtons
β	Slip parameter: Measures the degree of slip or non-slip behavior between a fluid and a solid surface.	Dimensionless
a	Carreau parameter: This is used in Carreau model to characterize the behavior of non-Newtonian fluids.	S
α	Inclined angle: It is a measure of the angle between a surface and the vertical or horizontal plane.	Degree (^{0}) or Rad (rad)

Table 4: Key terms in fluid dynamics.

Terms	Meaning	Significance
MHD	Magnetohydrodynamic	It studies the interaction between magnetic fields, fluids and the forces that acted upon them.
G, g	Gravitational force	It attracts two objects with mass towards each other.
M	Magnetic field	It describes the magnetic influence on the moving electrically conducting fluid.
Br	Brinkmann number	It is used to characterize the deformation of non-Newtonian fluids or used to predict the onset of non-Newtonian behavior in fluids.
PDE	Partial Differential Equation	This is used to model and analyze the fluid flow using the Navier-stokes equations (The continuity and the momentum equation), Euler equations.
ODE	Ordinary Differential Equation	This is used to model and analyze the fluid flow using Bernoulli's equation
PHF	Potential Heat Flow	Measure the heat transfer rate between a fluid and a surface.
PCF	Porous Conductivity Factor	Evaluate the flow resistance in a porous medium
We	Weissenberg number	It's a physical parameter used to determine the behavior of the flowing fluid.
KBM	Keller Box Method	It is a numerical technique used by N Nallagundla et al. to solve the derived ordinary differential equation.