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COMMON COUPLED FIXED POINTS FOR TWO PAIRS OF w-COMPATIBLE MAPS IN PARTIAL G-METRIC SPACES

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ABSTRACT

In this paper we prove a unique common coupled fixed point theorem for two pairs of w-compatible mappings satisfying two contractive conditions in partial G-metric spaces. We also furnish an example to support our main theorem.

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Keywords: Partial G-metric space, w-compatible pairs, 0-P-G completeness.

INTRODUCTION

Dhage [5] introduced the concept of $D-$metric spaces to generalize the ordinary metric spaces and proved several results, for example, refer [5, 6, 7]. Unfortunately almost all results are invalid (see [19, 20, 21, 13, 15]). To modify $D-$metric space, Mustafa and Sims [13] introduced the concept of $G-$metric spaces and obtained some results in their papers. Later several authors, for instance, [4, 10, 2, 3, 22, 24, 25, 26, 9, 14, 16, 17, 18], proved some fixed, common fixed and coupled fixed point theorems in $G-$metric spaces.

Recently Salimi and Vetro [23] defined partial $G-$metric spaces using the concept of partial metric spaces introduced by Mathews [12].

Kaewcharoen [10] proved a unique common fixed point theorem for four self mappings on a $G-$complete metric spaces. The intent of this paper is to extend the theorem of kaewcharoen [10] in partial $G-$metric spaces. We illustrated our theorem with an example.

First we state the following known definitions, lemmas and propositions.

Definition 1.1 [5]: Let X be a non-empty set. A $D-$metric on X is a function $D: X^3 \to [0,+\infty)$

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that satisfies the following conditions for each $x, y, z, a \in X$,

1. $D(x, y, z) = 0$ if and only if $x = y = z$,
2. $D(x, y, z) = D(p\{x, y, z\})$ where p is a permutation function,
3. $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z)$.

Then the pair (X, D) is called a D-metric space.

Definition 1.2 [13] : Let X be a non-empty set and let $G : X \times X \times X \to [0, \infty)$ be a function satisfying the following properties :

(G_1): $G(x, y, z) = 0$ if $x = y = z$,

(G_2): $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

(G_3): $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,

(G_4): $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$, symmetry in all three variables,

(G_5): $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then the function G is called a generalized metric or a G-metric on X and the pair (X, G) is called a G-metric space.

Definition 1.3 [12]: A partial metric on a non-empty set X is a function $p : X \times X \to [0, \infty)$ such that for all $x, y, z \in X$,

(p_1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

(p_2) $p(x, x) \leq p(x, y), p(y, y) \leq p(x, y)$,

(p_3) $p(x, y) = p(y, x)$,

(p_4) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$.

The pair (X, p) is called a *partial metric space* (PMS).

Definition 1.4 [23]: Let X be a non-empty set and let $P : X \times X \times X \to [0, +\infty)$ be called a partial G-metric if the following conditions are satisfied*:*

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(P1) If $x = y = z$ then $P(x, y, z) = P(x, x, x) = P(y, y, y) = P(z, z, z)$,

(P2) $P(x, x, x) + P(y, y, y) + P(z, z, z) \leq 3P(x, y, z)$ for all $x, y, z, \in X$,

(P3) $\frac{1}{3}P(x, x, x) + \frac{2}{3}P(y, y, y) < P(x, y, y)$ for all $x, y \in X$ with $x \neq y$,

(P4) $P(x, x, y) - \frac{1}{3}P(x, x, x) \leq P(x, y, z) - \frac{1}{3}P(z, z, z)$ for all points $x, y, z \in X$ with $y \neq z$,

(P5) $P(x, y, z) = P(x, z, y) = P(y, z, x) = \cdots$ (symmetry in three variables),

(P6) $P(x, y, z) \leq P(x, a, a) + P(a, y, z) - P(a, a, a)$ for any $x, y, z, a \in X$.

Then the pair (X, P) is called a partial G-metric space (in brief PGMS).

Example 1.1 [23]: Let $X = [0, +\infty)$ and define $P(x, y, z) = \frac{1}{3}(\max\{x, y\} + \max\{y, z\} + \max\{x, z\})$ for all points $x, y, z \in X$. Then (X, P) is a PGMS.

The following Proposition gives some properties of a partial G-metric.

Proposition 1.1 [23]: Let (X, P) be a PGMS. Then for $x, y, z, a \in X$, the following properties hold:

1. If $P(x, y, z) = P(x, x, x) = P(y, y, y) = P(z, z, z)$, then $x = y = z$
2. If $P(x, y, z) = 0$ then $x = y = z$;
3. If $x \neq y$, then $P(x, y, y) > 0$
4. $P(x, y, z) \leq P(x, x, y) + P(x, x, z) - P(x, x, x)$ for any $x, y, z, a \in X$.
5. $P(x, y, y) \leq 2P(x, x, y) - P(x, x, x)$;
6. $P(x, y, z) \leq P(x, a, a) + P(y, a, a) - P(z, a, a) - 2P(a, a, a)$;
7. $P(x, y, z) \leq P(x, a, z) + P(a, y, z) - \frac{2}{3}P(a, a, a) - \frac{1}{3}P(z, z, z)$ with $y \neq z$;

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8. $P(x,y,y) \le P(x,y,a) + P(a,y,y) - \frac{2}{3}P(a,a,a) - \frac{1}{3}P(y,y,y)$ with $x \ne y$;

Definition 1.5 [23]: Let (X,P) be a PGMS. Then

1. A sequence $\{x_n\}$ is $P-G$-convergent to $x \in X$ if and only if

$$P(x,x,x) = \lim_{n\to+\infty} P(x,x,x_n) = \lim_{n\to+\infty} P(x,x_n,x_n).$$

2. A sequence $\{x_n\}$ is $0-P-G$-Cauchy if and only if

$$\lim_{m,n\to+\infty} P(x_n,x_m,x_m) = 0.$$

3. A partial G-metric space (X,P) is said to be $0-P-G$-complete if and only if every $0-P-G$-Cauchy sequence in X $P-G$-converges to a point $x \in X$ such that $P(x,x,x)=0$.

Example 1.2 [23]: Let $X=[0,1]$ and $P: X^3 \to [0,\infty)$ be defined by $P(x,y,z) = \max\{x,y\} + \max\{y,z\} + \max\{x,z\}$ for all points $x,y,z \in X$. Then (X,P) is a $0-P-G-$ complete partial $G-$metric space.

Lemma 1.1 [23]: Let (X,P) be a partial G-metric space and $\{x_n\}$ be a sequence in X. Assume that $\{x_n\}$ $P-G$-converges to $x \in X$ and $P(x,x,x)=0$. Then $\lim_{n\to+\infty} P(x_n,y,y) = P(x,y,y)$ for all $y \in X$.

Similarly we can have the following Lemma.

Lemma 1.2: Let (X,P) be a partial G-metric space and $\{x_n\}$ be a sequence in X. Assume that $\{x_n\}$ $P-G$-converges to $x \in X$ and $P(x,x,x)=0$. Then $\lim_{n\to+\infty} P(x_n,x_n,y) = P(x,x,y)$ for all $y \in X$.

Bhskar and Lakshmikantham [8] developed some coupled fixed point theorems for a mapping satisfying mixed monotone property in partially ordered metric spaces. Later Lakshmikantham and Ciric [11] extended the notion of mixed monotone property to mixed g-monotone property and generalized the results of [8]. Abbas et al. [1] introduced $w-$compatible mappings and proved some common coupled fixed point theorems in cone metric spaces.

Definition 1.6 [8]: An element $(x,y) \in X \times X$ is called a coupled fixed point of a mapping $F: X \times X \to X$ if $x = F(x,y)$ and $y = F(y,x)$.

Definition 1.7 [11]: An element $(x,y) \in X \times X$ is called

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(i) a coupled coincident point of mappings $F : X \times X \to X$ and $f : X \to X$ if $fx = F(x, y)$ and $fy = F(y, x)$.

(ii) a common coupled fixed point of mappings $F : X \times X \to X$ and $f : X \to X$ if $x = fx = F(x, y)$ and $y = fy = F(y, x)$.

Definition 1.8 [1]: The mappings $F : X \times X \to X$ and $f : X \to X$ are called a w - compatible pair if $f(F(x, y)) = F(fx, fy)$ and $f(F(y, x)) = F(fy, fx)$ whenever $fx = F(x, y)$ and $fy = F(y, x)$.

In 2012, A.Kaewcharoen [10] proved the following

Theorem 1.1 (Theorem 2.1, [10]): Let X be a $G-$complete metric space. suppose that $\{f, S\}$ and $\{g, T\}$ are weakly compatible pairs of self-mappings on X satisfying

$$G(fx, fx, gy) \leq h \max\left\{\begin{matrix} G(Sx, Sx, Ty), G(fx, fx, Sx), G(gy, gy, Ty), \\ \frac{1}{2}(G(fx, fx, Ty) + G(gy, gy, Sx)) \end{matrix}\right\}$$

and

$$G(fx, gy, gy) \leq h \max\left\{\begin{matrix} G(Sx, Ty, Ty), G(fx, Sx, Sx), G(gy, Ty, Ty), \\ \frac{1}{2}(G(fx, Ty, Ty) + G(gy, Sx, Sx)) \end{matrix}\right\}$$

for all $x, y \in X$ where $h \in \left[0, \frac{1}{2}\right)$. Suppose that $fX \subseteq TX$ and $gX \subseteq SX$. If one of TX or SX is a $G-$closed subspace of X, then f, g, S and T have a unique common fixed point.

Now we give our main result.

MAIN RESULT

Theorem 2.1: *Let (X, P) be a partial G-metric space. Suppose that $f, g : X \times X \to X$ and $S, T : X \to X$ be satisfying*

(2.1.1) $f(X \times X) \subseteq T(X), g(X \times X) \subseteq S(X)$,

(2.1.2) $\{f, S\}$ and $\{g, T\}$ are w-compatible pairs,

(2.1.3) One of $T(X)$ or $S(X)$ is $0-P-G$-complete subspace of X,

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(2.1.4) $(a) P(f(x,y), f(x,y), g(u,v))$

$$\leq k\max\left\{\begin{array}{l} P(Sx,Sx,Tu), P(Sy,Sy,Tv), \\ P(f(x,y), f(x,y), Sx), P(f(y,x), f(y,x), Sy), \\ P(g(u,v), g(u,v), Tu), P(g(v,u), g(v,u), Tv), \\ \frac{1}{2}[P(f(x,y), f(x,y), Tu) + P(g(u,v), g(u,v), Sx)], \\ \frac{1}{2}[P(f(y,x), f(y,x), Tv) + P(g(v,u), g(v,u), Sy)] \end{array}\right\}$$

and

$(b) P(f(x,y), g(u,v), g(u,v))$

$$\leq k\max\left\{\begin{array}{l} P(Sx,Tu,Tu), P(Sy,Tv,Tv), \\ P(f(x,y), Sx, Sx), P(f(y,x), Sy, Sy), \\ P(g(u,v), Tu, Tu), P(g(v,u), Tv, Tv), \\ \frac{1}{2}[P(f(x,y), Tu, Tu) + P(g(u,v), Sx, Sx)], \\ \frac{1}{2}[P(f(y,x), Tv, Tv) + P(g(v,u), Sy, Sy)] \end{array}\right\}$$

for all $x, y, u, v \in X$, where $k \in [0, \frac{1}{2})$.

Then f, g, S and T have a unique common coupled fixed point in $X \times X$.

Proof: Let $(x_0, y_0) \in (X \text{ x } X)$. From (2.1.1), we can construct the sequences $\{x_n\}, \{y_n\}, \{z_n\}$ and $\{w_n\}$ such that

$$f(x_{2n}, y_{2n}) = Tx_{2n+1} = z_{2n},$$

$$f(y_{2n}, x_{2n}) = Ty_{2n+1} = w_{2n},$$

$$g(x_{2n+1}, y_{2n+1}) = Sx_{2n+2} = z_{2n+1},$$

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$$g(y_{2n+1}, x_{2n+1}) = Sy_{2n+2} = w_{2n+1},\ n = 0\ ,\ 1\ ,.2$$

Now from $(2.1.4)(b)$, we have

$$P(z_{2n+1}, z_{2n+1}, z_{2n}) = P(g(x_{2n+1}, y_{2n+1}), g(x_{2n+1}, y_{2n+1}), f(x_{2n}, y_{2n}))$$

$$\leq k \max \begin{Bmatrix} P(z_{2n-1}, z_{2n}, z_{2n}), P(w_{2n-1}, w_{2n}, w_{2n}), \\ P(z_{2n}, z_{2n-1}, z_{2n-1}), P(w_{2n}, w_{2n-1}, w_{2n-1}), \\ P(z_{2n+1}, z_{2n}, z_{2n}), P(w_{2n+1}, w_{2n}, w_{2n}), \\ \frac{1}{2}[P(z_{2n}, z_{2n}, z_{2n}) + P(z_{2n+1}, z_{2n-1}, z_{2n-1})], \\ \frac{1}{2}[P(w_{2n}, w_{2n}, w_{2n}) + P(w_{2n+1}, w_{2n-1}, w_{2n-1})] \end{Bmatrix}$$

$$\leq k \max \begin{Bmatrix} P(z_{2n-1}, z_{2n}, z_{2n}), P(w_{2n-1}, w_{2n}, w_{2n}), \\ 2P(z_{2n}, z_{2n}, z_{2n-1}), 2P(w_{2n}, w_{2n}, w_{2n-1}), \\ 2P(z_{2n+1}, z_{2n+1}, z_{2n}), 2P(w_{2n+1}, w_{2n+1}, w_{2n}), \\ \frac{1}{2}[2P(z_{2n+1}, z_{2n+1}, z_{2n}) + 2P(z_{2n}, z_{2n}, z_{2n-1})], \\ \frac{1}{2}[2P(w_{2n+1}, w_{2n+1}, w_{2n}) + 2P(w_{2n}, w_{2n}, w_{2n-1})] \end{Bmatrix},$$

from (P_6) *and Proposition* (1.1 *v*)

$$= 2k \max \begin{Bmatrix} P(z_{2n-1}, z_{2n}, z_{2n}), P(z_{2n+1}, z_{2n+1}, z_{2n}), \\ P(w_{2n-1}, w_{2n}, w_{2n}), P(w_{2n+1}, w_{2n+1}, w_{2n}) \end{Bmatrix} \tag{2.1}$$

Similarly we can prove,

$$P(w_{2n+1}, w_{2n+1}, w_{2n}) \leq 2k \max \begin{Bmatrix} P(z_{2n-1}, z_{2n}, z_{2n}), \\ P(z_{2n+1}, z_{2n+1}, z_{2n}), \\ P(w_{2n-1}, w_{2n}, w_{2n}), \\ P(w_{2n+1}, w_{2n+1}, w_{2n}) \end{Bmatrix} \tag{2.2}$$

Thus from (2.1) and (2.2), we have

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$$\max\left\{\begin{array}{l}P(z_{2n+1},z_{2n+1},z_{2n}),\\P(w_{2n+1},w_{2n+1},w_{2n})\end{array}\right\}\leq 2k\max\left\{\begin{array}{l}P(z_{2n-1},z_{2n},z_{2n}),\\P(z_{2n+1},z_{2n+1},z_{2n}),\\P(w_{2n-1},w_{2n},w_{2n}),\\P(w_{2n+1},w_{2n+1},w_{2n})\end{array}\right\} \tag{2.3}$$

Now suppose that

$\max\{P(z_{2n},z_{2n},z_{2n-1}),P(w_{2n},w_{2n},w_{2n-1})\}=0$.

Then we have $z_{2n-1}=z_{2n}$ and $w_{2n-1}=w_{2n}$ from Proposition 1.1(ii)

From (2.3),

$$\max\{P(z_{2n+1},z_{2n+1},z_{2n}),P(w_{2n+1},w_{2n+1},w_{2n})\}=0 \tag{2.4}$$

so that $z_{2n}=z_{2n+1}$ and $w_{2n}=w_{2n+1}$.

Now from $(2.1.4)(a)$, we can prove

$$P(z_{2n+2},z_{2n+2},z_{2n+1})\leq 2k\max\left\{\begin{array}{l}P(z_{2n+1},z_{2n+1},z_{2n}),\\P(z_{2n+2},z_{2n+2},z_{2n+1}),\\P(w_{2n+1},w_{2n+1},w_{2n}),\\P(w_{2n+1},w_{2n+2},w_{2n+2})\end{array}\right\} \tag{2.5}$$

and

$$P(w_{2n+2},w_{2n+2},w_{2n+1})\leq 2k\max\left\{\begin{array}{l}P(z_{2n+1},z_{2n+1},z_{2n}),\\P(z_{2n+2},z_{2n+2},z_{2n+1}),\\P(w_{2n+1},w_{2n+1},w_{2n}),\\P(w_{2n+1},w_{2n+2},w_{2n+2})\end{array}\right\} \tag{2.6}$$

Thus from (2.5) and (2.6), we have

$$\max\left\{\begin{array}{l}P(z_{2n+2},z_{2n+2},z_{2n+1}),\\P(w_{2n+2},w_{2n+2},w_{2n+1})\end{array}\right\}\leq 2k\max\left\{\begin{array}{l}P(z_{2n+1},z_{2n+1},z_{2n}),\\P(z_{2n+2},z_{2n+2},z_{2n+1}),\\P(w_{2n+1},w_{2n+1},w_{2n}),\\P(w_{2n+2},w_{2n+2},w_{2n+1})\end{array}\right\} \tag{2.7}$$

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Using (2.4) in (2.7), we get

$$\max\{P(z_{2n+2}, z_{2n+2}, z_{2n+1}), P(w_{2n+2}, w_{2n+2}, w_{2n+1})\} = 0$$

so that $z_{2n+2} = z_{2n+1}$ and $w_{2n+2} = w_{2n+1}$.

Continuing in this way we get $z_{2n} = z_{2n+1} = z_{2n+2} = \cdots$ and $w_{2n} = w_{2n+1} = w_{2n+2} = \cdots$

Thus $\{z_n\}$ and $\{w_n\}$ are Cauchy sequences.

Assume that $\max\{P(z_{n+1}, z_{n+1}, z_n), P(w_{n+1}, w_{n+1}, w_n)\} > 0$ for all n.

Now from (2.3) and (2.7) we have

$$\begin{aligned}\max\begin{Bmatrix} P(z_{n+1}, z_{n+1}, z_n), \\ P(w_{n+1}, w_{n+1}, w_n) \end{Bmatrix} &\le 2k \max\begin{Bmatrix} P(z_{n-1}, z_n, z_n), \\ P(w_{n-1}, w_n, w_n) \end{Bmatrix} \\ &\le (2k)^2 \max\begin{Bmatrix} P(z_{n-2}, z_{n-1}, z_{n-1}), \\ P(w_{n-2}, w_{n-2}, w_{n-1}) \end{Bmatrix} \\ &\;. \\ &\;. \\ &\;. \\ &\le (2k)^n \max\begin{Bmatrix} P(z_0, z_1, z_1), \\ P(w_0, w_1, w_1) \end{Bmatrix}\end{aligned}$$

Thus

$$\lim_{n\to\infty} P(z_n, z_{n+1}, z_{n+1}) = 0 \tag{2.8}$$

and

$$\lim_{n\to\infty} P(w_n, w_{n+1}, w_{n+1}) = 0 \tag{2.9}$$

For $m,n \in N$ with $m > n$, we have

$$P(z_n, z_m, z_m) \le P(z_n, z_{n+1}, z_{n+1}) + P(z_{n+1}, z_{n+2}, z_{n+2}) + \cdots + P(z_{m-1}, z_m, z_m)$$

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$$\leq [(2k)^n + (2k)^{n+1} + \ldots + (2k)^{m-1}] \max \begin{Bmatrix} P(z_0, z_1, z_1), \\ P(w_0, w_1, w_1) \end{Bmatrix}$$

$$\leq \frac{(2k)^n}{1-2k} \max \begin{Bmatrix} P(z_0, z_1, z_1), \\ P(w_0, w_1, w_1) \end{Bmatrix}$$

Thus

$$\lim_{n,m\to\infty} p(z_n, z_m, z_m) = 0 \tag{2.10}$$

Similarly, we have

$$\lim_{n,m\to\infty} p(w_n, w_m, w_m) = 0 \tag{2.11}$$

Thus $\{z_n\}$ and $\{w_n\}$ are $0-P-G-$ Cauchy sequences in X.

Suppose $S(X)$ is $0-P-G$ complete. Then the sequences $\{z_{2n+1}\} = \{Sx_{2n+2}\}$ and $\{w_{2n+1}\} = \{Sy_{2n+2}\}$ $P-G$ converge to points $\alpha, \beta \in S(X)$ such that $p(\alpha,\alpha,\alpha) = 0$ and $P(\beta,\beta,\beta) = 0$ and $\alpha = Su$ and $\beta = Sv$ for some $u, v \in X$.

Since $\{z_n\}$ and $\{w_n\}$ are $0-P-G-$ Cauchy and from (2.8) and (2.9), it follows that $\{z_{2n}\}$ and $\{w_{2n}\}$ are $P-G-$ converge to α and β respectively.

Using $(2.1.4)(b)$, we obtain that

$$P(z_{2n+1}, z_{2n+1}, f(u,v)) = P(g(x_{2n+1}, y_{2n+1}), g(x_{2n+1}, y_{2n+1}), f(u,v))$$

$$\leq k \max \begin{Bmatrix} P(Su, z_{2n}, z_{2n}), P(Sv, w_{2n}, w_{2n}), \\ P(f(u,v), Su, Su), P(f(v,u), Sv, Sv), \\ P(z_{2n+1}, z_{2n}, z_{2n}), P(w_{2n+1}, w_{2n}, w_{2n}), \\ \frac{1}{2}[P(f(u,v), z_{2n}, z_{2n}) + P(z_{2n+1}, Su, Su)], \\ \frac{1}{2}[P(f(v,u), w_{2n}, w_{2n}) + P(w_{2n+1}, Sv, Sv)] \end{Bmatrix}$$

Letting $n \to \infty$ *we have*

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$$P(\alpha,\alpha, f(u,v)) \le k \max\begin{Bmatrix} 0,0, P(f(u,v),\alpha,\alpha), P(f(v,u),\beta,\beta), \\ \frac{1}{2}[P(f(u,v),\alpha,\alpha)+P(\alpha,\alpha,\alpha)], \\ \frac{1}{2}[P(f(v,u),\beta,\beta)+P(\beta,\beta,\beta)] \end{Bmatrix}$$

from (2.8), (2.9), Lemmas (1.1) and (1.2)

$$= k \max\{P(f(u,v),\alpha,\alpha), P(f(v,u),\beta,\beta)\} \tag{2.12}$$

Using $(2.1.4)(b)$ to $P(w_{2n+1}, w_{2n+1}, f(v,u))$ and then letting $n \to \infty$, we get

$$P(\beta,\beta, f(v,u)) \le k \max\{P(f(u,v),\alpha,\alpha), P(f(v,u),\beta,\beta)\} \tag{2.13}$$

Thus from (2.12) and (2.13) we have

$$\max\begin{Bmatrix} P(\alpha,\alpha, f(u,v)), \\ P(\beta,\beta, f(v,u)) \end{Bmatrix} \le k \max\{P(f(u,v),\alpha,\alpha), P(f(v,u),\beta,\beta)\}$$

which in turn yields from Proposition 1.1(ii) that $f(u,v)=\alpha=Su$ and $f(v,u)=\beta=Sv$. Thus (α,β) is a coupled coincidence point of f and S. Since $\{f,S\}$ is a w-compatible pair, we have $S\alpha = f(\alpha,\beta)$ and $S\beta = f(\beta,\alpha)$.

We next prove that $S\alpha=\alpha$ and $S\beta=\beta$.

Applying $(2.1.4)(b)$, we obtain that

$$P(z_{2n+1}, z_{2n+1}, S\alpha) = P(g(x_{2n+1}, y_{2n+1}), g(x_{2n+1}, y_{2n+1}), f(\alpha,\beta))$$

$$\le k \max\begin{Bmatrix} P(S\alpha, z_{2n}, z_{2n}), P(S\beta, w_{2n}, w_{2n}), \\ P(f(\alpha,\beta), S\alpha, S\alpha), P(f(\beta,\alpha), S\beta, S\beta), \\ P(z_{2n+1}, z_{2n}, z_{2n}), P(w_{2n+1}, w_{2n}, w_{2n}), \\ \frac{1}{2}[P(f(\alpha,\beta), z_{2n}, z_{2n}) + P(z_{2n+1}, S\alpha, S\alpha)], \\ \frac{1}{2}[P(f(\beta,\alpha), w_{2n}, w_{2n}) + P(w_{2n+1}, S\beta, S\beta)] \end{Bmatrix}.$$

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Taking $n \to \infty$, we have

$$P(\alpha,\alpha,S\alpha) \le k \max \left\{ \begin{array}{l} P(S\alpha,\alpha,\alpha), P(S\beta,\beta,\beta), \\ P(S\alpha,S\alpha,S\alpha), P(S\beta,S\beta,S\beta), 0, 0, \\ \frac{1}{2}[P(S\alpha,\alpha,\alpha)+P(\alpha,S\alpha,S\alpha)], \\ \frac{1}{2}[P(S\beta,\beta,\beta)+P(\beta,S\beta,S\beta)] \end{array} \right\}$$

from lemma (1.1), (1.2) and (2.8), (2.9)

$$\le k \max \left\{ \begin{array}{l} P(S\alpha,\alpha,\alpha), P(S\beta,\beta,\beta), \\ P(S\alpha,\alpha,\alpha)+P(\alpha,S\alpha,S\alpha), \\ P(S\beta,\beta,\beta)+P(\beta,S\beta,S\beta), \\ \frac{1}{2}[P(S\alpha,\alpha,\alpha)+P(\alpha,S\alpha,S\alpha)], \\ \frac{1}{2}[P(S\beta,\beta,\beta)+P(\beta,S\beta,S\beta)] \end{array} \right\}$$

$$\le k \max \left\{ \begin{array}{l} P(S\alpha,\alpha,\alpha)+P(\alpha,S\alpha,S\alpha), \\ P(S\beta,\beta,\beta)+P(\beta,S\beta,S\beta) \end{array} \right\} \quad (2.14)$$

Using $(2.1.4)(b)$ to $P(w_{2n+1}, w_{2n+1}, S\beta)$ and then letting $n \to \infty$, we get

$$P(\beta,\beta,S\beta) \le k \max \left\{ \begin{array}{l} P(S\alpha,\alpha,\alpha)+P(\alpha,S\alpha,S\alpha), \\ P(S\beta,\beta,\beta)+P(\beta,S\beta,S\beta) \end{array} \right\} \quad (2.15)$$

From (2.14) and (2.15), we have

$$\max \left\{ \begin{array}{l} P(\alpha,\alpha,S\alpha), \\ P(\beta,\beta,S\beta) \end{array} \right\} \le k \max \left\{ \begin{array}{l} P(S\alpha,\alpha,\alpha)+P(\alpha,S\alpha,S\alpha), \\ P(S\beta,\beta,\beta)+P(\beta,S\beta,S\beta) \end{array} \right\}$$

$$\le k \left[\begin{array}{l} \max\{P(S\alpha,\alpha,\alpha), P(S\beta,\beta,\beta)\} \\ +\max\{P(\alpha,S\alpha,S\alpha), P(\beta,S\beta,S\beta)\} \end{array} \right]$$

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Thus

$$\max\begin{Bmatrix} P(\alpha,\alpha,S\alpha), \\ P(\beta,\beta,S\beta) \end{Bmatrix} \le \frac{k}{1-k}\max\begin{Bmatrix} P(\alpha,S\alpha,S\alpha), \\ P(\beta,S\beta,S\beta) \end{Bmatrix} \tag{2.16}$$

Using $(2.1.4)(a)$, we have

$$P(z_{2n+1},S\alpha,S\alpha) = P(g(x_{2n+1},y_{2n+1}), f(\alpha,\beta), f(\alpha,\beta))$$

$$\le k\max\begin{Bmatrix} P(S\alpha,S\alpha,z_{2n}), P(S\beta,S\beta,w_{2n}), \\ P(S\alpha,S\alpha,S\alpha), P(S\beta,S\beta,S\beta), \\ P(z_{2n+1},z_{2n+1},z_{2n}), P(w_{2n+1},w_{2n+1},w_{2n}), \\ \frac{1}{2}[P(S\alpha,S\alpha,z_{2n}) + P(z_{2n+1},z_{2n+1},S\alpha)], \\ \frac{1}{2}[P(S\beta,S\beta,w_{2n}) + P(w_{2n+1},w_{2n+1},S\beta)] \end{Bmatrix}.$$

Taking $n \to \infty$, we have

$$P(\alpha,S\alpha,S\alpha) \le k\max\begin{Bmatrix} P(S\alpha,S\alpha,\alpha), P(S\beta,S\beta,\beta), \\ P(S\alpha,\alpha,\alpha) + P(\alpha,S\alpha,S\alpha), \\ P(S\beta,\beta,\beta) + P(\beta,S\beta,S\beta),0,0, \\ \frac{1}{2}[P(S\alpha,S\alpha,\alpha) + P(\alpha,\alpha,S\alpha)], \\ \frac{1}{2}[P(S\beta,S\beta,\beta) + P(\beta,\beta,S\beta)] \end{Bmatrix}$$

$$= k\max\begin{Bmatrix} P(S\alpha,\alpha,\alpha) + P(\alpha,S\alpha,S\alpha), \\ P(S\beta,\beta,\beta) + P(\beta,S\beta,S\beta) \end{Bmatrix} \tag{2.17}$$

Applying $(2.1.4)(a)$ to $P(w_{2n+1},S\beta,S\beta)$ and then letting $n \to \infty$, we get

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$$P(\beta, S\beta, S\beta) \le k \max\left\{\begin{matrix} P(S\alpha,\alpha,\alpha) + P(\alpha, S\alpha, S\alpha), \\ P(S\beta,\beta,\beta) + P(\beta, S\beta, S\beta) \end{matrix}\right\} \quad (2.18)$$

From (2.17) and (2.18), we have

$$\max\left\{\begin{matrix} P(\alpha, S\alpha, S\alpha), \\ P(\beta, S\beta, S\beta) \end{matrix}\right\} \le k \max\left\{\begin{matrix} P(S\alpha,\alpha,\alpha) + P(\alpha, S\alpha, S\alpha), \\ P(S\beta,\beta,\beta) + P(\beta, S\beta, S\beta) \end{matrix}\right\}$$

$$\le k\left[\begin{matrix} \max\{P(S\alpha,\alpha,\alpha), P(S\beta,\beta,\beta)\} \\ + \max\{P(\alpha, S\alpha, S\alpha), P(\beta, S\beta, S\beta)\} \end{matrix}\right]$$

Thus

$$\max\left\{\begin{matrix} P(\alpha, S\alpha, S\alpha), \\ P(\beta, S\beta, S\beta) \end{matrix}\right\} \le \frac{k}{1-k} \max\left\{\begin{matrix} P(S\alpha,\alpha,\alpha), \\ P(S\beta,\beta,\beta) \end{matrix}\right\} \quad (2.19)$$

From (2.16) and (2.19), we have

$$\max\left\{\begin{matrix} P(\alpha, \alpha, S\alpha), \\ P(\beta, \beta, S\beta) \end{matrix}\right\} \le \left(\frac{k}{1-k}\right)^2 \max\left\{\begin{matrix} P(S\alpha,\alpha,\alpha), \\ P(S\beta,\beta,\beta) \end{matrix}\right\}$$

so that $S\alpha = \alpha$ and $S\beta = \beta$. Thus $\alpha = S\alpha = f(\alpha,\beta)$ and $\beta = S\beta = f(\beta,\alpha)$. Since $f(X \times X) \subseteq T(X)$, there exist $a, b \in X$ such that $\alpha = f(\alpha,\beta) = Ta$ and $\beta = f(\beta,\alpha) = Tb$.

From $(2.1.4)(a)$ we obtain

$$P(\alpha,\alpha, g(a,b)) = P(f(\alpha,\beta), f(\alpha,\beta), g(a,b))$$

$$\le k \max\left\{\begin{matrix} P(\alpha,\alpha,\alpha), P(\beta,\beta,\beta), \\ P(\alpha,\alpha,\alpha), P(\beta,\beta,\beta), \\ P(g(a,b), g(a,b), \alpha), P(g(b,a), g(b,a), \beta), \\ \frac{1}{2}[P(\alpha,\alpha,\alpha) + P(g(a,b), g(a,b), \alpha)], \\ \frac{1}{2}[P(\beta,\beta,\beta) + P(g(b,a), g(b,a), \beta)] \end{matrix}\right\}$$

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$$\leq 2k \max\begin{Bmatrix} P(g(a,b),\alpha,\alpha), \\ P(g(b,a),\beta,\beta) \end{Bmatrix} \tag{2.20}$$

Again using $(2.1.4)(a)$ to $P(\beta,\beta,g(b,a))$,we obtain

$$P(\beta,\beta,g(b,a)) \leq 2k \max\begin{Bmatrix} P(g(a,b),\alpha,\alpha), \\ P(g(b,a),\beta,\beta) \end{Bmatrix} \tag{2.21}$$

From (2.20) and (2.21),we have

$$\max\begin{Bmatrix} P(\alpha,\alpha,g(a,b)), \\ P(\beta,\beta,g(b,a)) \end{Bmatrix} \leq k \max\begin{Bmatrix} P(\beta,\beta,g(b,a)), \\ P(\alpha,\alpha,g(a,b)) \end{Bmatrix}$$

so that $g(a,b)=\alpha=Ta$ and $g(b,a)=\beta=Tb$. Since the pair $\{g,T\}$ is weakly compatible, we have $T\alpha=g(\alpha,\beta)$ and $T\beta=g(\beta,\alpha)$. Now we prove $T\alpha=\alpha$ and $T\beta=\beta$.

Using $(2.1.4)(a)$ we obtain

$$P(\alpha,\alpha,T\alpha)=P(f(\alpha,\beta),f(\alpha,\beta),g(\alpha,\beta))$$

$$\leq k \max\begin{Bmatrix} P(\alpha,\alpha,T\alpha),P(\beta,\beta,T\beta), \\ P(\alpha,\alpha,\alpha),P(\beta,\beta,\beta), \\ P(T\alpha,T\alpha,T\alpha),P(T\beta,T\beta,T\beta), \\ \frac{1}{2}[P(\alpha,\alpha,T\alpha)+P(T\alpha,T\alpha,\alpha)], \\ \frac{1}{2}[P(\beta,\beta,T\beta)+P(T\beta,T\beta,\beta)] \end{Bmatrix}$$

$$\leq k \max\begin{Bmatrix} P(\alpha,\alpha,T\alpha),P(\beta,\beta,T\beta),0,0, \\ P(T\alpha,\alpha,\alpha)+P(\alpha,T\alpha,T\alpha), \\ P(T\beta,\beta,\beta)+P(\beta,T\beta,T\beta), \\ \frac{1}{2}[P(\alpha,\alpha,T\alpha)+P(T\alpha,T\alpha,\alpha)], \\ \frac{1}{2}[P(\beta,\beta,T\beta)+P(T\beta,T\beta,\beta)] \end{Bmatrix}$$

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$$= k \max\left\{\begin{array}{l} P(T\alpha,\alpha,\alpha)+P(\alpha,T\alpha,T\alpha), \\ P(T\beta,\beta,\beta)+P(\beta,T\beta,T\beta) \end{array}\right\} \quad (2.22)$$

Similarly from (2.1.4)(a), we obtain

$$P(\beta,\beta,T\beta) \le k \max\left\{\begin{array}{l} P(T\alpha,\alpha,\alpha)+P(\alpha,T\alpha,T\alpha), \\ P(T\beta,\beta,\beta)+P(\beta,T\beta,T\beta) \end{array}\right\} \quad (2.23)$$

From (2.22) and (2.23)

$$\max\left\{\begin{array}{l} P(\alpha,\alpha,T\alpha), \\ P(\beta,\beta,T\beta) \end{array}\right\} \le k \max\left\{\begin{array}{l} P(T\alpha,\alpha,\alpha)+P(\alpha,T\alpha,T\alpha), \\ P(T\beta,\beta,\beta)+P(\beta,T\beta,T\beta) \end{array}\right\}$$

$$\le k\left[\begin{array}{l} \max\{P(T\alpha,\alpha,\alpha),P(T\beta,\beta,\beta)\} \\ +\max\{P(\alpha,T\alpha,T\alpha),P(\beta,T\beta,T\beta)\} \end{array}\right]$$

Thus

$$\max\left\{\begin{array}{l} P(\alpha,\alpha,T\alpha), \\ P(\beta,\beta,T\beta) \end{array}\right\} \le \frac{k}{1-k} \max\left\{\begin{array}{l} P(\alpha,T\alpha,T\alpha), \\ P(\beta,T\beta,T\beta) \end{array}\right\} \quad (2.24)$$

Now Using $(2.1.4)(b)$ as in above, we obtain

$$\max\left\{\begin{array}{l} P(T\alpha,T\alpha,\alpha), \\ P(T\beta,T\beta,\beta) \end{array}\right\} \le \frac{k}{1-k} \max\left\{\begin{array}{l} P(\alpha,T\alpha,T\alpha), \\ P(\beta,T\beta,T\beta) \end{array}\right\} \quad (2.25)$$

From (2.24) and (2.25), we have

$$\max\left\{\begin{array}{l} P(\alpha,\alpha,T\alpha), \\ P(\beta,\beta,T\beta) \end{array}\right\} \le \left(\frac{k}{1-k}\right)^2 \max\left\{\begin{array}{l} P(\alpha,T\alpha,T\alpha), \\ P(\beta,T\beta,T\beta) \end{array}\right\}$$

so that $\alpha = T\alpha$ and $\beta = T\beta$. Thus $\alpha = T\alpha = g(\alpha,\beta)$ and $\beta = T\beta = g(\beta,\alpha)$. Hence (α,β) is a common coupled fixed point of f,g,S and T.

Suppose that $(\alpha^1,\beta^1) \in X \times X$ is another common coupled fixed point of f,g,S and T.

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Suppose that $\alpha \neq \alpha^1$ and $\beta \neq \beta^1$.

Applying $(2.1.4)(a)$, we obtain that

$$P(\alpha,\alpha,\alpha^1) = P(f(\alpha,\beta), f(\alpha,\beta), g(\alpha^1,\beta^1))$$

$$\leq k \max \left\{ \begin{array}{l} P(\alpha,\alpha,\alpha^1), P(\beta,\beta,\beta^1), \\ P(\alpha,\alpha,\alpha), P(\beta,\beta,\beta), \\ P(\alpha^1,\alpha^1,\alpha^1), P(\beta^1,\beta^1,\beta^1), \\ \frac{1}{2}\left[P(\alpha,\alpha,\alpha^1) + P(\alpha^1,\alpha^1,\alpha)\right], \\ \frac{1}{2}\left[P(\beta,\beta,\beta^1) + P(\beta^1,\beta^1,\beta)\right] \end{array} \right\}$$

$$\leq k \max \left\{ \begin{array}{l} P(\alpha,\alpha,\alpha^1), P(\beta,\beta,\beta^1), 0, 0, \\ P(\alpha^1,\alpha,\alpha) + P(\alpha,\alpha^1,\alpha^1), \\ P(\beta^1,\beta,\beta)) + P(\beta,\beta^1,\beta^1), \\ \frac{1}{2}\left[P(\alpha,\alpha,\alpha^1) + P(\alpha^1,\alpha^1,\alpha)\right], \\ \frac{1}{2}\left[P(\beta,\beta,\beta^1) + P(\beta^1,\beta^1,\beta)\right] \end{array} \right\},$$

from (P_6)

$$= k \max \left\{ \begin{array}{l} P(\alpha^1,\alpha,\alpha) + P(\alpha,\alpha^1,\alpha^1), \\ P(\beta^1,\beta,\beta) + P(\beta,\beta^1,\beta^1) \end{array} \right\} \tag{2.26}$$

Again using $(2.1.4)(a)$, we obtain

$$P(\beta,\beta,\beta^1) \leq k \max \left\{ \begin{array}{l} P(\alpha^1,\alpha,\alpha) + P(\alpha,\alpha^1,\alpha^1), \\ P(\beta^1,\beta,\beta) + P(\beta,\beta^1,\beta^1) \end{array} \right\} \tag{2.27}$$

From (2.26) and (2.27), we have

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$$\max\begin{Bmatrix} P(\alpha,\alpha,\alpha^1), \\ P(\beta,\beta,\beta^1) \end{Bmatrix} \le k \max\begin{Bmatrix} P(\alpha^1,\alpha,\alpha)+P(\alpha,\alpha^1,\alpha^1), \\ P(\beta^1,\beta,\beta)+P(\beta,\beta^1,\beta^1) \end{Bmatrix}$$

$$\le k\begin{bmatrix} \max\{P(\alpha^1,\alpha,\alpha),P(\beta^1,\beta,\beta)\} \\ +\max\{P(\alpha,\alpha^1,\alpha^1),P(\beta,\beta^1,\beta^1)\} \end{bmatrix}$$

so that

$$\max\begin{Bmatrix} P(\alpha,\alpha,\alpha^1), \\ P(\beta,\beta,\beta^1) \end{Bmatrix} \le \frac{k}{1-k} \max\begin{Bmatrix} P(\alpha,\alpha^1,\alpha^1), \\ P(\beta,\beta^1,\beta^1) \end{Bmatrix} \tag{2.28}$$

Similarly applying (2.1.4)(b) to $P(\alpha^1,\alpha^1,\alpha)$ and $P(\beta^1,\beta^1,\beta)$, we obtain that

$$\max\begin{Bmatrix} P(\alpha^1,\alpha^1,\alpha), \\ P(\beta^1,\beta^1,\beta) \end{Bmatrix} \le \frac{k}{1-k} \max\begin{Bmatrix} P(\alpha,\alpha,\alpha^1), \\ P(\beta,\beta,\beta^1) \end{Bmatrix} \tag{2.29}$$

From (2.28) and (2.29), we have

$$\max\begin{Bmatrix} P(\alpha,\alpha,\alpha^1), \\ P(\beta,\beta,\beta^1) \end{Bmatrix} \le \left(\frac{k}{1-k}\right)^2 \max\begin{Bmatrix} P(\alpha,\alpha,\alpha^1), \\ P(\beta,\beta,\beta^1) \end{Bmatrix} \tag{2.30}$$

so that $\alpha=\alpha^1$ and $\beta=\beta^1$. Thus (α,β) is the unique common coupled fixed point of f,g,S and T.

If we put $f=g$ and $S=T$ in Theorem 2.1, we have the following Corollary.

Corollary 2.1 *Let* (X,P) *be a partial* G*-metric space. Suppose that* $f:X\times X\to X$ *and* $S:X\to X$ *be satisfying*

(2.1.1) $f(X\times X)\subseteq T(X)$,

(2.1.2) (f,T) are weakly compatible pairs,

(2.1.3) $T(X)$ is $0-P-G$-complete subspace of X,

(2.1.4) $(a)P(f(x,y),f(x,y),f(u,v))$

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$$\leq k\max\left\{\begin{array}{l} P(Tx,Tx,Tu), P(Ty,Ty,Tv), \\ P(f(x,y), f(x,y),Tx), P(f(y,x), f(y,x),Ty), \\ P(f(u,v), f(u,v),Tu), P(f(v,u), f(v,u),Tv), \\ \frac{1}{2}[P(f(x,y), f(x,y),Tu)+P(f(u,v), f(u,v),Tx)], \\ \frac{1}{2}[P(f(y,x), f(y,x),Tv)+P(f(v,u), f(v,u),Ty)] \end{array}\right\}$$

and

$(b) P(f(x,y), f(u,v), f(u,v))$

$$\leq k\max\left\{\begin{array}{l} P(Tx,Tu,Tu), P(Ty,Tv,Tv), \\ P(f(x,y),Tx,Tx), P(f(y,x),Ty,Ty), \\ P(f(u,v),Tu,Tu), P(f(v,u),Tv,Tv), \\ \frac{1}{2}[P(f(x,y),Tu,Tu)+P(f(u,v),Tx,Tx)], \\ \frac{1}{2}[P(f(y,x),Tv,Tv)+P(f(v,u),Ty,Ty)] \end{array}\right\}$$

for all $x, y, u, v \in X$, where $k \in [0, \frac{1}{2})$.

Then f and T have a unique common coupled fixed point in $X \times X$.

Now we give the following example to illustrate our Theorem 2.1

Example 2.1 *Let* (X, P) *be a partial* G*-metric space, where* $X = [0,1]$ $P : X \times X \times X \to [0, \infty)$ be defined by

$P(x, y, z) = max\{x, y\} + max\{y, z\} + max\{x, z\}$.

Let $f, g : X \times X \to X$ and $S, T : X \to X$ be defined by

$$f(x,y) = \frac{x^2 + y^2}{16},\ g(x,y) = \frac{x+y}{32},\ Sx = \frac{x^2}{2},\ Tx = \frac{x}{4}, \forall\ x, y \in X.$$

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The conditions $(2.1.1)$, $(2.1.2)$ and $(2.1.3)$ are obvious.

For all $x, y \in X$, consider

$$P(f(x,y), f(x,y), g(u,v)) = f(x,y) + 2\max\{f(x,y), g(u,v)\}$$

$$= \frac{x^2+y^2}{16} + 2\max\left\{\frac{x^2+y^2}{16}, \frac{u+v}{32}\right\}$$

$$= \left[\frac{x^2}{16} + 2\max\left\{\frac{x^2}{16}, \frac{u}{32}\right\}\right] + \left[\frac{y^2}{16} + 2\max\left\{\frac{y^2}{16}, \frac{v}{32}\right\}\right]$$

$$= \frac{1}{8}\left[\frac{x^2}{2} + 2\max\left\{\frac{x^2}{2}, \frac{u}{4}\right\}\right] + \frac{1}{8}\left[\frac{y^2}{2} + 2\max\left\{\frac{y^2}{2}, \frac{v}{4}\right\}\right]$$

$$= \frac{1}{8}[Sx + 2\max\{Sx, Ty\}] + \frac{1}{4}[Sy + 2\max\{Sy, Tv\}]$$

$$= \frac{1}{8}[P(Sx, Sx, Tu) + P(Sy, Sy, Tv)]$$

$$= \frac{1}{8}[P(Sx, Sx, Tu) + P(Sy, Sy, Tv)]$$

$$= \frac{1}{8}[P(Sx, Sx, Tu) + P(Sy, Sy, Tv)]$$

$$\leq \frac{1}{4}\max\{P(Sx, Sx, Tu), P(Sy, Sy, Tv)\}$$

$$\leq \frac{1}{4}\max\left\{\begin{array}{l} P(Sx, Sx, Tu), P(Sy, Sy, Tv), \\ P(f(x,y), f(x,y), Sx), P(f(y,x), f(y,x), Sy), \\ P(g(u,v), g(u,v), Tu), P(g(v,u), g(v,u), Tv), \\ \frac{1}{2}[P(f(x,y), f(x,y), Tu) + P(g(u,v), g(u,v), Sx)], \\ \frac{1}{2}[P(f(y,x), f(y,x), Tv) + P(g(v,u), g(v,u), Sy)] \end{array}\right\},$$

One can easily verify $(2.1.4)(b)$ in the similar lines.

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Thus all conditions of Theorem (2.1) are satisfied. Clearly $(0,0)$ is the unique common coupled fixed point of f, g, S and T.

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