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THE STUDY OF VARIATION OF COEFFICIENT OF RESTITUTION ON SUBJECTING THE BOUNCING SURFACE WITH A4 PAPER

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ABSTRACT

The coefficient of restitution (COR) of two colliding objects is the ratio of the relative velocity of separation to the relative velocity of approach. The coefficient of restitution varies from materials chosen. In this research we are interested in studying the variation of the coefficient of restitution by adding A4 paper on the bouncing surface, where bouncing surface is the base for the ball bounce (example, thick plane plywood). We, then study the trend of the observed data i.e. we look at the nature of coefficient of restitution over the increasing number of A4 paper.

Keywords: variation of coefficient of restitution, bouncing surface

INTRODUCTION

The coefficient of restitution [1] (COR, represented by the letter *e*) is the study of the nature of collision. A value of e = 1 represents a perfectly elastic interaction while the value of *e* = 0 indicates that the interaction is totally inelastic. Measuring the COR and other properties has been the subject of great interest in the field of sports. Many researchers have invested their efforts to identify the easier method of determining the coefficient of restitution while some researchers have studied the other properties. For example: COR as a fluctuating quantity [2], behavior of bouncing balls, in various specific sport applications [3] etc. The method of varying coefficient of restitution has not been studied in a great detail, may be due to its main application is in sports only.

In this article, we are presenting the outcome of our experiment that we have performed in a simple home experiment. We are interested to check whether or not the coefficient of restitution varies from its initial value on modifying the surface of bounce. We performed one dimensional collision of a bouncing ball on a heavy flat surface over plywood. Then we gradually added A4 papers in increasing order of number on the plywood surface. We assumed that when the number of papers increases, the contact with respect to the original surface varies and thus the coefficient of restitution also varies. The A4 paper that we have used is easily accessible in market and it's a product of Bindals Papers Mills Limited, India. While the plywood (thickness 3.1cm) we used as a bouncing surface is obtained from SHAAN Furniture and Industries, Banepa Kavre.

METHODOLOGY

Since, we are only interested in finding the nature or trend of graph between the increasing thickness of A4 paper and value of COR; we only need the theory to calculate the coefficient of restitution. There have been many publications made on various method of determining coefficient of restitution [4-13]. Here we use the technique as described in [4] and [12]. As explained in Methodology by A. Wadhwa in [12], we use the same equation $e = \Delta t \sqrt{\frac{g}{8h}}$ where Δt is the time interval between for one complete bounce, g is the acceleration due to gravity, and h is the height from where the ball is dropped freely. We used the open source software called Audacity which records the sounds facilitating us to interpret and analyze the time intervals the recorded sound.

At first, we tried the experiment by simple addition of 10 papers on the bouncing surface (plywood) and calculated the coefficient of restitution for series of experiment like on adding next 10 papers, and so on. But doing this without any press on the paper, we found rapid decrement in the value of COR and thus we could hardly get 5 data i.e up to 50 added papers. And after that it became hard for us to calculate coefficient of restitution because it was very difficult to calculate the time period from the recorded image on Audacity software. So we decided to press a little bit with 10 kg constant weight and finally obtained more data. This variation has occurred due to the spaces between the papers which fill air molecules and hence causing a rapid decrement in the value of COR. This might conclude that there is variation due to pressing on such paper but we are only interested in finding the trend of decrement of COR on such set up (if possible). Therefore, we used constant 10 kg weight to press the A4 paper so that we could neglect the air filled space occurring between the papers and obtained the data by increment of 10 papers per experiment up to 130 papers on average. Some of the recordings of images that are obtained from Audacity, where we calculated time period are shown below in figure.
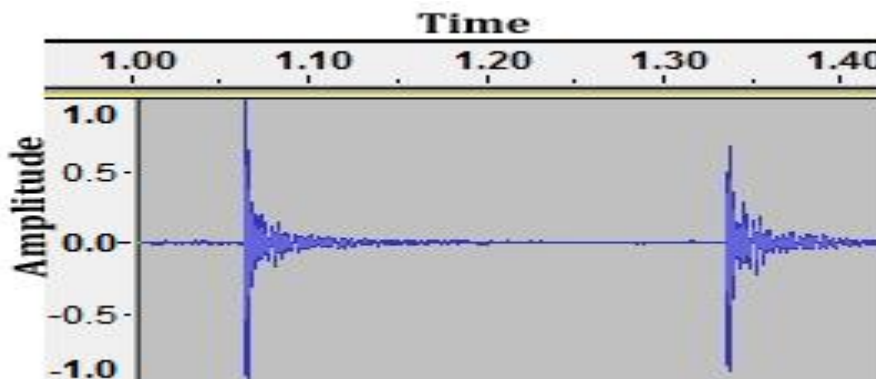

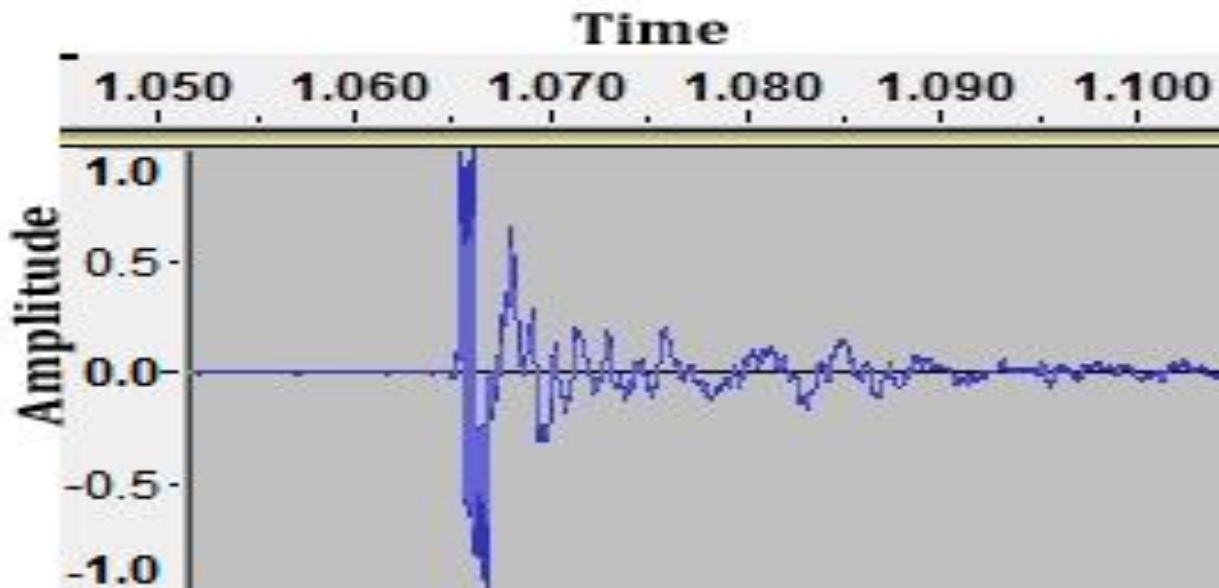Figure 1 Audacity image of recorded sound of bounce
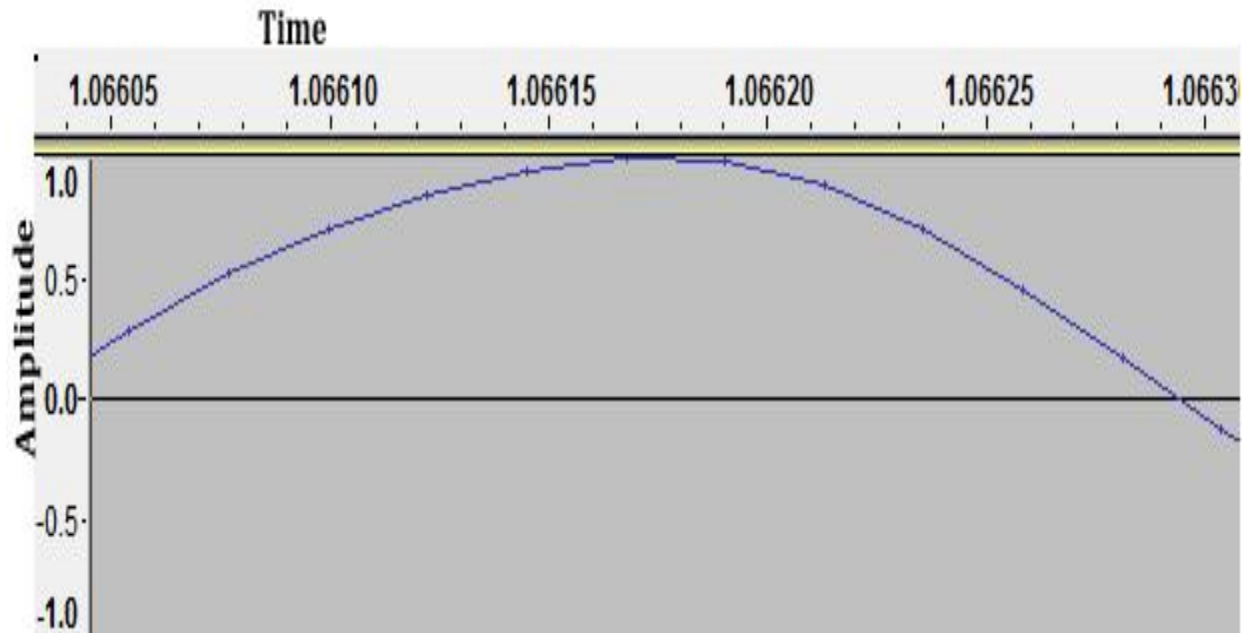
Figure 2 Zoomed audacity image of fig. 1

Figure 3 Zoomed image of fig.2

As shown in above figure, by zooming each images, the software has capability to show the zoomed version of time as well due to which we can accurately measure the time interval by choosing the highest peak as an average.

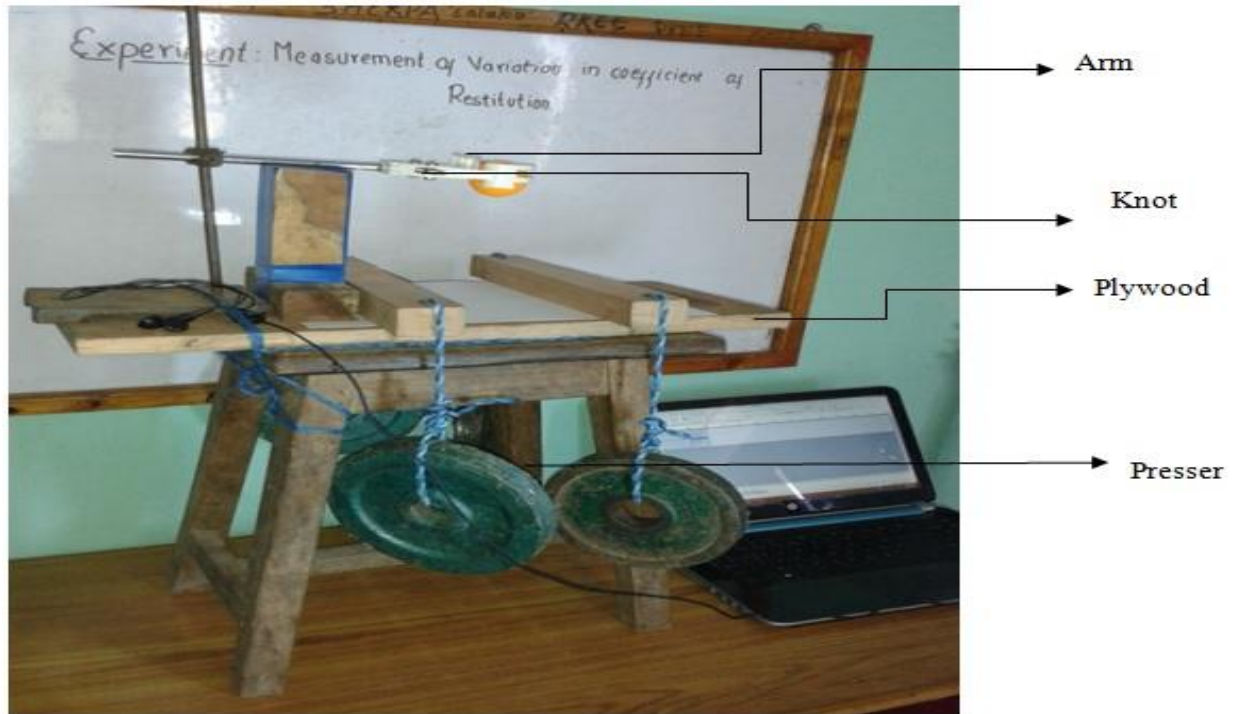Also we have used the following apparatus for our calculation.

Figure 4 Experimental Setup

As labelled in the image above, the base is a ply wood of thicknes 3.1 cm which was available locally. And the arm is of the stand which is common stand of many physics, chemistry, biology lab. The given ball is allowed to fall vertically best without sliping by loosening the knot. Also we have used Presser in order to press the paper so as to avoid air molecules, which increases the number of data that can be interpreted in Audacity software. We performed experiment with table tenis ball (Double Fish) and a rubber ball on the same surface of plywood.The images of those used materials are shown in image below.

RESULT &

Figure 5 TT ball with label maker   Figure 6 Rubber ball

DISCUSSION

As discussed in methodology, we have finally calculated the time period which in turn helps us to calculate the actual value of coefficient of restitution. The ball was bounced a number of times and then we took the average of all in order to get accuracy in our calculations. We however, could not measure all the values of time. Because as number of paper increased, the time period was decreasing and was becoming lower. Our software plotted a single wave form where it was difficult to identify the time differences of highest peak. So we only calculated those values where the software could distinctly show the two different highest peaks.

For the calculation of thickness of a single A4 paper (surface density=70 gsm) a bundle of papers containing known numbers were taken. The average width of the bundle was measured and was divided by the number of papers in the bundle. This experiment was performed a number of times and the average thickness of single paper was found to be 0.089mm. The following table shows the experimental data on calculation of thickness of paper.

Paper Quantity	Experiment number					Average thickness on 1 paper
	1	2	3	4	5	
	thickness(mm)	thickness(mm)	thickness(mm)	thickness(mm)	thickness(mm)	
10	0.89	0.91	0.91	0.87	0.88	0.0892
20	1.75	1.75	1.71	1.75	1.75	0.0871
30	2.65	2.66	2.67	2.69	2.65	0.0888
40	3.56	3.58	3.56	3.6	3.6	0.0895
Average of average (mm)						0.089

Table 1 The calculation for average thickness of single A4 paper

The average of time and the value of coefficient of restitution over different thickness of paper for a T.T ball when dropped from 14.6cm height were obtained as:

T	T square	Coefficient of Restitution	Number of papers	Thickness in mm
0.31758	0.100857	0.919908773	0	0
0.28986	0.084019	0.83961445	10	0.89
0.26732	0.07146	0.774324621	20	1.78
0.2674	0.071503	0.774556351	30	2.67
0.2493	0.06215	0.722127518	40	3.56
0.23046	0.053112	0.667555186	50	4.45
0.22028	0.048523	0.638067588	60	5.34
0.21024	0.044201	0.608985517	70	6.23
0.22258	0.049542	0.644729815	80	7.12
0.18714	0.035021	0.542073581	90	8.01
0.17362	0.030144	0.50291127	100	8.9
0.16764	0.028103	0.485589479	110	9.79

0.16334	0.02668	0.47313401	120	10.68
0.15764	0.02485	0.456623273	122	10.858
0.15396	0.023704	0.445963709	124	11.036

Table 2 Observed average value of coefficient of restitution with TT ball over 14.6 cm height on plywood

Here, the height was 14.6 cm and initial surface was taken as the wooden plywood. The above data were plotted in software called Origin. The observed data showed the trend such that the value of coefficient of restitution was decreasing with increasing thickness. But as thickness increases, the value of coefficient of restitution decreases. We could not measure all the value of coefficient of restitution over larger value of thickness because Audacity software was unable to distinguish the separate peaks as shown in figure 1. However, we could see the ball bouncing even over wide range of thickness of 7.12 cm (approximately 800 papers). Observing the above experiment we decided to use the fitting tool in Origin with the exponential function as $y = y_0 + A_1 e^{-\frac{x}{t_1}}$. By setting y_0 equals zero and substituting A_1 as the value of coefficient of restitution calculated without any addition of A4 paper on the surface of plywood or marble. Here, x is the thickness of added paper and $1/t_1$ is the decay constant. The negative sign indicates the value of coefficient of restitution is decreasing over thickness. The fitted curve with above condition and function is shown below.
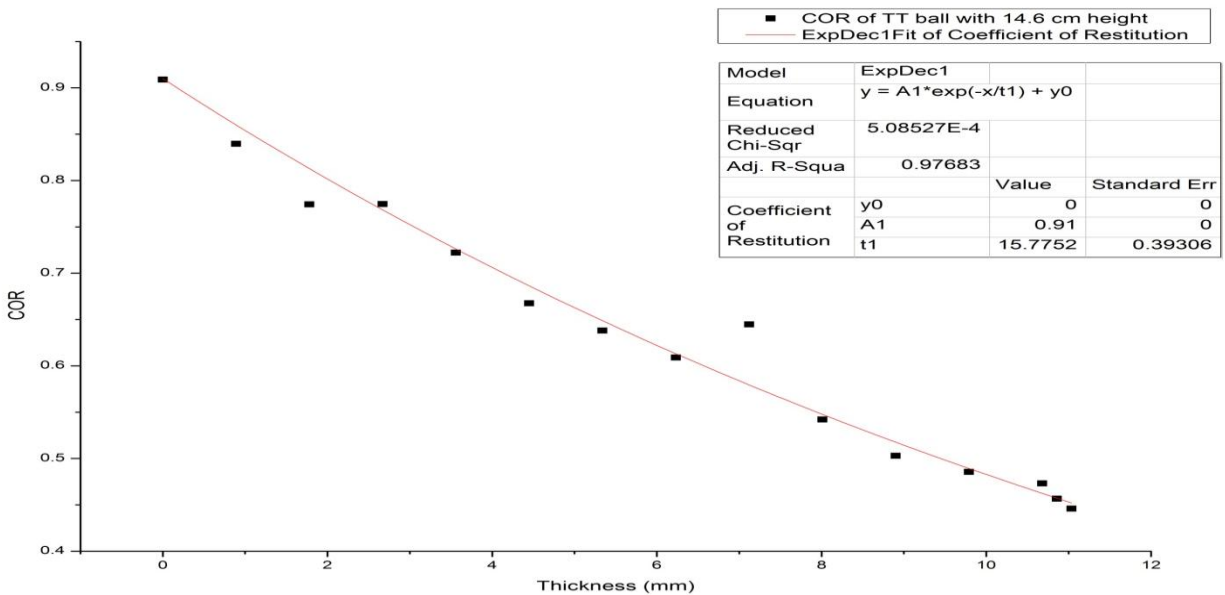

Figure 7 Best fitted curve with the help of Origin of data shown in Table 2

We again changed the height to 17.9 cm and calculated the coefficient of restitution. The calculated values of COR (Coefficient of Restitution) as shown in table below:

T	T^2	COR	Number of papers	Thickness
0.3412	0.116417	0.892587	0	0
0.32532	0.105833	0.851045	10	0.89
0.31396	0.098571	0.821327	20	1.78
0.30318	0.091918	0.793126	30	2.67
0.2807	0.078792	0.734318	40	3.56
0.25618	0.065628	0.670173	50	4.45
0.26028	0.067746	0.680898	60	5.34
0.23368	0.054606	0.611312	70	6.23
0.23622	0.0558	0.617957	80	7.12
0.21706	0.047115	0.567834	90	8.01
0.20576	0.042337	0.538273	100	8.9
0.17784	0.031627	0.465233	110	9.79
0.16818	0.028285	0.439963	120	10.68
0.15654	0.024505	0.409512	130	11.57
0.1555	0.02418	0.406792	134	11.926
0.15692	0.024624	0.410506	138	12.282

Table 3 Observed value of COR with 17.9 cm height of TT ball over plywood

And these values of COR is again plotted over thickness and using best curve fit, we calculated approximately close value of decay constant with previous results. In the data shown in Table 2, the value of decay constant($1/t_1$) was obtained as 0.0634 (as shown in figure 7) with value of t_1 as 15.77 while for the data presented in Table 3, the value of decay constant was obtained as 0.0609 with value of t_1 as 16.40 as shown in figure 8. The best fitted graph for data in Table 3 is shown below
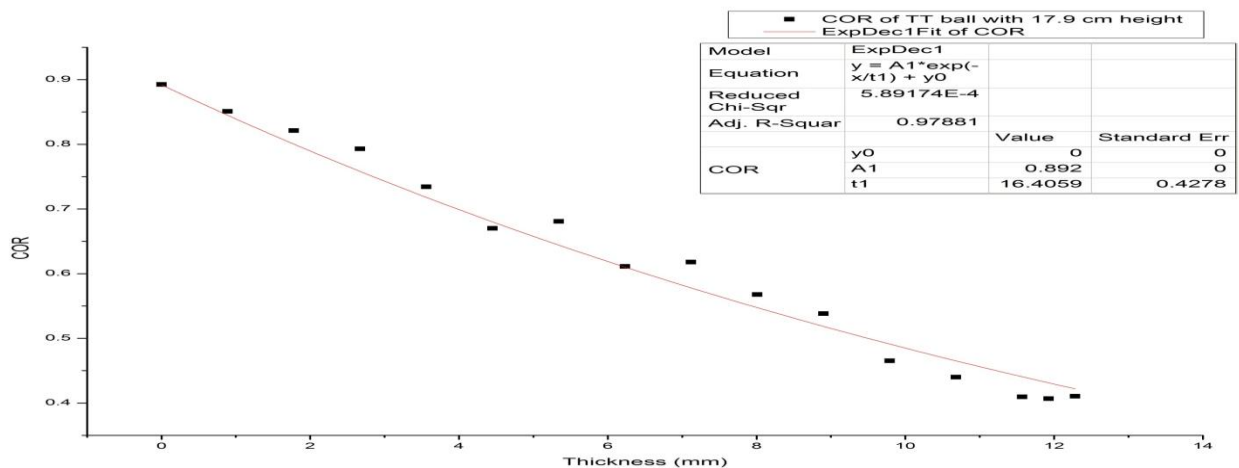

Figure 8 Best fitted curve of data in Table 3

We also assumed that this might become true only for TT ball we have chosen. So, we selected rubber ball shown in figure 5. Over the same plywood surface, we used the rubber ball and the value of coefficient of restitution were obtained for the height 14.6 cm height as

T	T^2	COR	Number of papers	Thickness
0.281933	0.079486	0.816653904	0	0
0.2706	0.073224	0.783825537	20	1.78
0.2662	0.070862	0.771080406	40	3.56
0.245133	0.06009	0.710058266	60	5.34
0.222633	0.049566	0.644884302	80	7.12
0.2391	0.057169	0.692581988	100	8.9
0.2381	0.056692	0.689685367	120	10.68
0.217867	0.047466	0.631077077	140	12.46
0.197767	0.039112	0.572855003	160	14.24
0.1725	0.029756	0.499667055	180	16.02

Table 4 Observed value of COR with 14.6 cm height of rubber ball over plywood

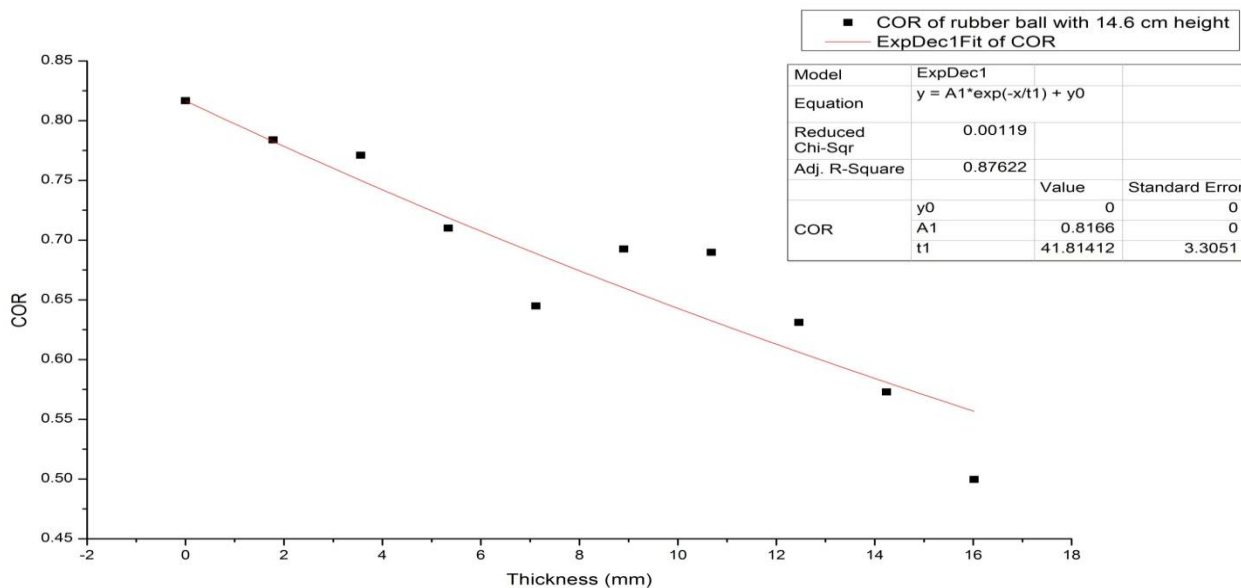Using the same fitting tool, the graph of COR with thickness was obtained as:

Figure 9 Best fitted curve for data in Table 4

Here we obtained the value of decay constant as 0.023 with value of t_1 as 41.81 (figure 9). Again, we repeated the experiment with same arrangements except we increased the height to 17.9 cm and the observed values of COR over different thickness were shown in table below

T	T^2	COR	Number of papers	Thickness
0.298633	0.089182	0.781232	0	0
0.278133	0.077358	0.727603	20	1.78
0.2775	0.077006	0.725946	40	3.56
0.2754	0.075845	0.720453	60	5.34
0.2562	0.065638	0.670225	80	7.12
0.262833	0.069081	0.687578	100	8.9
0.240667	0.05792	0.629589	120	10.68
0.211	0.044521	0.551981	140	12.46
0.19965	0.03986	0.522289	160	14.24
0.189533	0.035923	0.495824	180	16.02

Table 5 Observed value of COR with 17.9 cm height of rubber ball over plywood

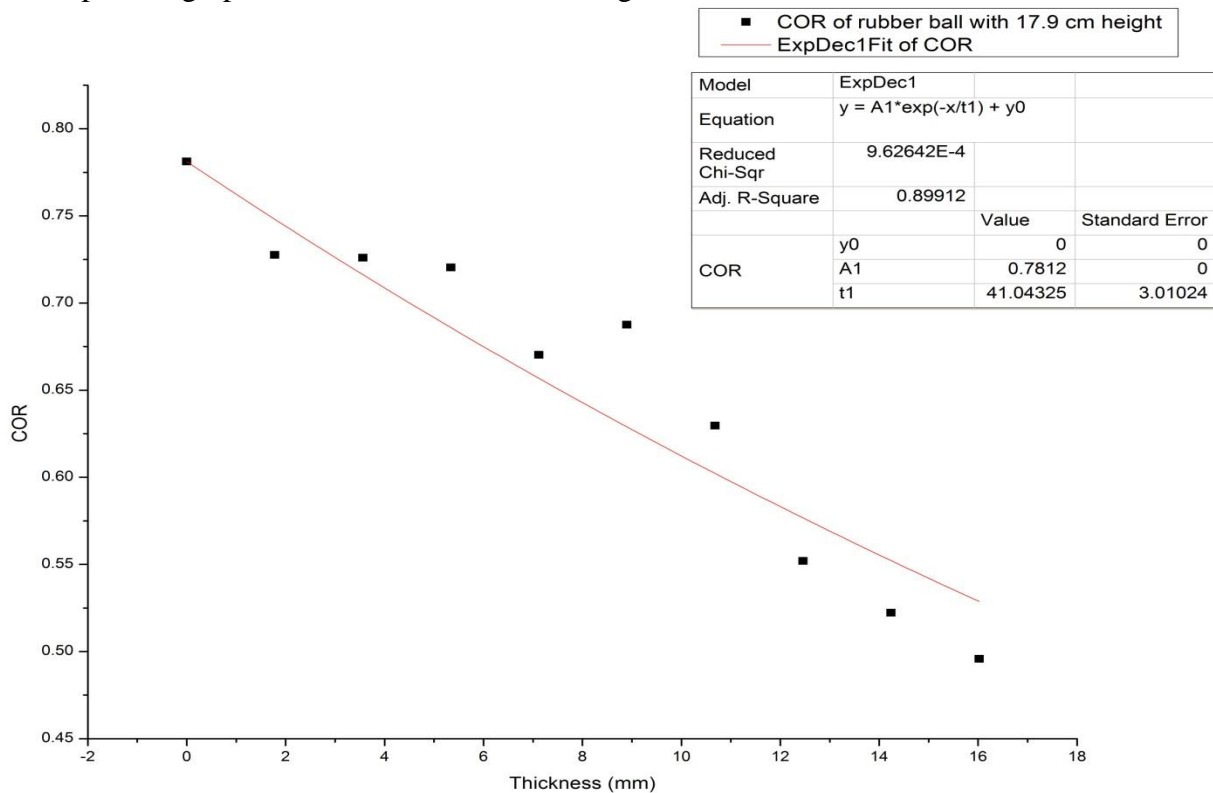The respective graph of COR with thickness along with best fitted curve is shown below

Figure 10 Best fitted curve of data obtained from Table 5

As seen from the above data, the decay constant observed for this experiment was 0.024 with value of t_1 as 41.04 (figure 10) which again were close to the decay constant with that of rubber ball of 14.6 cm height.

CONCLUSION

In conclusion, we verified our assumption that the coefficient of restitution would decrease while subjecting bounce surface with increasing number of A4 papers. We obtained almost similar values of decay constant for TT ball and rubber ball over bounce from different height. The decay constant for TT ball when dropped freely from 14.6cm height was 0.0634 and from height 17.9cm, decay constant was 0.0609. Similarly for rubber ball, the value of decay constant obtained for height 14.6 cm was 0.023 while for the height 17.9cm the decay constant was 0.024. This implies that for one set of system, we get its own trend of decay. The decay trend varies from material to material. In this experiment, decay trend was almost same over different height for TT ball. Also, decay trend was similar over different height for rubber ball. But the decay trend of TT ball was totally different from rubber ball. With this experiment, we found one simple way or method of decreasing or varying the coefficient of restitution and with the advantage that this method showed some mathematical decreasing trend.

ACKNOWLEDGEMENT

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