TIME DEPENDENT TEMPERATURE DISTRIBUTION MODEL IN LAYERED HUMAN DERMAL PART

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ABSTRACT
The paper developed application of finite element method with linear function in the study of temperature distribution in the layers of dermal part–stratum corneum, stratum germinativum, papillary region, reticular region and subcutaneous tissues as elements. The method is applied to obtain the numerical solution of governing differential equation for one dimensional unsteady state bio-heat transfer using suitable values of parameters that effect the heat transfer in human body. The numerical results obtained are exhibited graphically for various atmospheric temperatures for comparative study of temperature distribution profiles. The loss of heat from the outer surface of the body to the environment is taken due to convection, radiation and sweat evaporation.

Keywords: finite element method, human dermal part, bio-heat equation

INTRODUCTION
Human skin is a very complex tissue consisting of several distinct layers and components. Hence it exhibits complex material behavior. It is the largest organ of the human body and it has several functions. The most obvious function is protection of the body against external influences and it helps to regulate the body temperature. Skin temperature distribution of the human body is a complex interaction of physical heat exchange processes and the potential for physiological adjustment. Temperature influences the functioning of biological systems. For humans, the core temperature varies within narrow bounds around 37°C. The human body acts like an unsteady state system with a continuous input and not a regular output. This continuous input provokes a mass storage in the body, leading for a reaction in order to burn it. This process is called metabolism rate. The normal range of human body temperature varies due to metabolic rate of an individual. Higher metabolic rate causes higher normal body temperature and lower metabolic rate causes lower the normal body temperature. Other factors that might affect the body temperature of an individual may be the time of day or the part of body where the temperature is measured. The body temperature is lower in the morning due to the resting condition and higher at night after a day of muscular activity and after food intake as well as the thermoregulatory processes of the body are carried on by different methods: convection, conduction, radiation and evaporation. Most of the heat of the body is lost due to convection because it is in contact with fluid such as air and water. Convective transfer by blood plays a key role. Densities of blood vessels are very high and hence it is hard to compute a detailed temperature distribution in even a small part of the body while accounting for all of the blood vessels individually. Fortunately, the effects of the blood vessels can be described collectively with some success by the help of Pennes bio heat equation.

So external (climate) and internal (metabolic) heat sources influences body temperature. Heavy exercise, illness and not hot and humid but also cold and windy environments alter body temperature outside the normal range. Ambient temperature, humidity, air movement and radiant heat from the sun as well as warm and cold surfaces contribute to climate heat stress. Body temperature reflects the careful balance between heat production and heat loss. There is a continuous heat exchange between the body and the environment. Bio- directional routes for heat exchange are: convection (Cv), conduction (Cd), and radiation (R). There are also two uni-directional routes: Metabolic heat (M) increases the thermal load, evaporation (E) decreases the load. The net heat storage (S) formula is:

$$S= M+/- Cv+/-Cd+/-R-E$$

When the net heat storages (S) is positive, body temperature will rise and when S is negative, it will go down.

The temperature model on human body deals the study of temperature distributions on the layers of human skin exposed to sources of temperature, e.g. surrounding temperature or any sources. So any irregularities in the temperature distribution in dermal layers due to abnormal environment cause the disturbances in thermoregulation. Hence the study of temperature distribution has the clinical and theoretical importance. Temperature plays an important role in the functioning of biological system. To predict tissue temperatures as well as the temperature of the skin layers influence of blood flow must be accounted. The temperature dependence of biological processes can be used to clinical effect. For example, hyperthermia treatment against cancer, cooling of the head to prevent hair loss as a side effect of chemotherapy and cooling of patients during major surgery to protect the brain are such examples.

Mathematical model for temperature distributions have a role both in treatment and diagnosis. They can aid in predicting the time in the course of treatment or giving information on the temperature where thermometry is lacking. So numerical modeling and analysis of the skin has numerous applications in medicine but also in aesthetics, in computer, graphic, etc.

MATHEMATICAL MODEL AND MODEL ASSUMPTIONS

The mathematical model used for bioheat transfer is based on the Pennes equation [8] which incorporates the effect of metabolism and blood perfusion in to the standard thermal diffusion equation. This equation is written in simplified form as

$$\rho c \frac{\partial T}{\partial t} = \nabla(K.\nabla T) + M(T_a - T) + S \qquad (1)$$

where,

ρ = Tissue density

c = Tissue specific heat

K = Tissue thermal conductivity
m_b = Blood mass flow rate
c_b = Blood specific heat
ω_b = The blood perfusion rate
T_a = Arterial blood temperature
M = $\omega_b\, m_b c_b$

S = Metabolic heat generation rate

The loss of heat from the skin surface due to convection, radiation and evaporation is considered. So the mixed boundary condition is

$$-K \left.\frac{\partial T}{\partial x}\right|_{\text{Skin surface}} = h\,(T - T_\infty) + LE \qquad (2)$$

where, h = Combined heat transfer coefficient due to convection and radiation

T_∞ = Surrounding temperature

L = Latent heat of evaporation

E = Rate of sweat evaporation

The inner body core temperature T_b is assumed to be 37^0C.

The thickness of stratum corneum, stratum germinativum, papillary region, reticular region and subcutaneous tissue have been considered as l_1, $l_2 - l_1$, $l_3 - l_2$, $l_4 - l_3$, $l_5 - l_4$ respectively and T_0, T_1, T_2 T_3, T_4 and $T_5 = T_b$ are the nodal temperatures at a distances $x = 0$, $x = l_1$, $x = l_2$, $x = l_3$, $x = l_4$ and $x = l_5$. $T^{(i)}$, i = 1, 2, 3, 4, 5 be the temperature functions in the layers stratum corneum, stratum germinativum, papillary region, reticular region and subcutaneous tissue respectively.

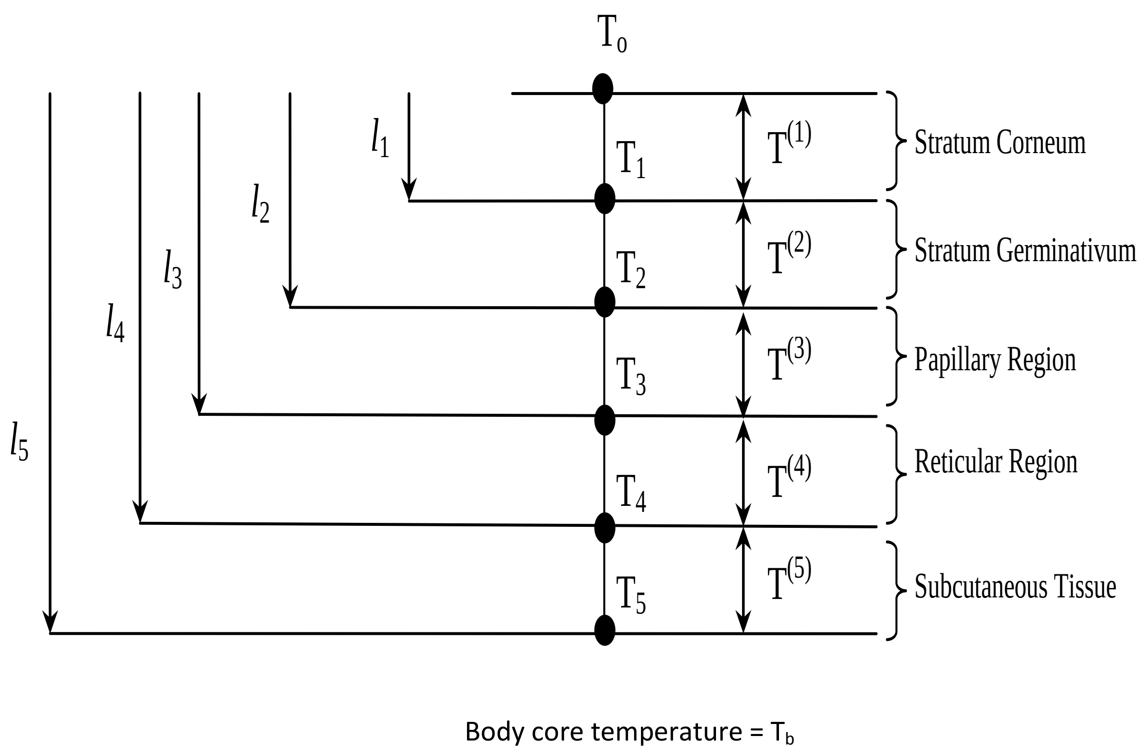

Figure 1: Schematic diagram of five layers of dermal part with nodal points

The anatomical structure of human dermal part makes it reasonable to consider M and S zero in stratum corneum. In the model, the thermal conductivity in the layers of dermal part is considered as constant. All the assumptions for parameters in the layers of dermal part can be summed up as:

Quantity	T	T_a	K	M	S
Outer Boundary	T_0	–	–	–	–
Stratum Corneum ($0\le x\le l_1$)	$T^{(1)} = T_0 + \frac{T_1 - T_0}{l_1}x$	$T_a^{(1)} = 0$	$K^{(1)}$	$M^{(1)} = 0$	$S^{(1)} = 0$
Stratum Germinativum ($l_1\le x\le l_2$)	$T^{(2)} = \frac{l_2T_1 - l_1T_2}{l_2 - l_1} + \frac{T_2 - T_1}{l_2 - l_1}x$	$T_a^{(2)} = 0$	$K^{(2)}$	$M^{(1)} = 0$	$S^{(2)} = \left(\frac{x - l_1}{l_4 - l_1}\right)s$
Papillary Region ($l_2\le x\le l_3$)	$T^{(3)} = \frac{l_3T_2 - l_2T_3}{l_3 - l_2} + \frac{T_3 - T_2}{l_3 - l_2}x$	$T_a^{(3)} = T_b$	$K^{(3)}$	$M^{(3)} = \left(\frac{x - l_2}{l_4 - l_2}\right)m$	$S^{(3)} = \left(\frac{x - l_1}{l_4 - l_1}\right)s$
Reticular Region ($l_3\le x\le l_4$)	$T^{(4)} = \frac{l_4T_3 - l_3T_4}{l_4 - l_3} + \frac{T_4 - T_3}{l_4 - l_3}x$	$T_a^{(4)} = T_b$	$K^{(4)}$	$M^{(4)} = \left(\frac{x - l_2}{l_4 - l_2}\right)m$	$S^{(4)} = \left(\frac{x - l_1}{l_4 - l_1}\right)s$
Subcutaneous Tissue ($l_4\le x\le l_5$)	$T^{(5)} = \frac{l_5T_4 - l_4T_5}{l_5 - l_4} + \frac{T_5 - T_4}{l_5 - l_4}x$	$T_a^{(5)} = T_b$	$K^{(5)}$	$M^{(5)} = m$	$S^{(5)} = s$

SOLUTION OF THE PROBLEM

The variational integral form of (1) in one dimensional unsteady state case together with outer skin boundary condition (2) is given by

$$I = \frac{1}{2}\int_0^l \left[K\left(\frac{dT}{dx}\right)^2 + M(T_a - T)^2 - 2ST + \rho c\left(\frac{\partial T^2}{\partial t}\right)\right]dx + \frac{1}{2}h(T - T_\infty)^2 + 2LET$$

We write I separately for the Five layers; I_1 for stratum corneum, I_2 for stratum germinativum, I_3 for papillary region, I_4 for reticular region and I_5 for subcutaneous tissue, so

$$I_1 = \frac{1}{2}\int_0^{l_1}\left[K^{(1)}\left(\frac{dT^{(1)}}{dx}\right)^2 + M^{(1)}\left(T_A - T^{(1)}\right)^2 - 2S^{(1)}T^{(1)} + \rho c\frac{\partial T^{(1)^2}}{\partial t}\right]dx + \frac{1}{2}\left[h(T_0 - T_a)^2 + 2LET_0\right]$$

$$I_2 = \frac{1}{2}\int_{l_1}^{l_2}\left[K^{(2)}\left(\frac{dT^{(2)}}{dx}\right)^2 + M^{(2)}\left(T_A - T^{(2)}\right)^2 - 2S^{(2)}T^{(2)} + \rho c\frac{\partial T^{(2)^2}}{\partial t}\right]dx$$

$$I_3 = \frac{1}{2}\int_{l_2}^{l_3}\left[K^{(3)}\left(\frac{dT^{(3)}}{dx}\right)^2 + M^{(3)}\left(T_A - T^{(3)}\right)^2 - 2S^{(3)}T^{(3)} + \rho c\frac{\partial T^{(3)^2}}{\partial t}\right]dx$$

$$I_4 = \frac{1}{2}\int_{l_3}^{l_4}\left[K^{(4)}\left(\frac{dT^{(4)}}{dx}\right)^2 + M^{(4)}\left(T_A - T^{(4)}\right)^2 - 2S^{(4)}T^{(4)} + \rho c\frac{\partial T^{(4)^2}}{\partial t}\right]dx$$

$$I_5 = \frac{1}{2}\int_{l_4}^{l_5}\left[K^{(5)}\left(\frac{dT^{(5)}}{dx}\right)^2 + M^{(5)}\left(T_A - T^{(5)}\right)^2 - 2S^{(5)}T^{(5)} + \rho c\frac{\partial T^{(5)^2}}{\partial t}\right]dx$$

Evaluating the integral I_i , i = 1, 2,…,5 with the help of layers wise assumptions, we get the following system of equations given below

$$I_1 = A_1 + B_1T_0 + D_1T_0^2 + E_1T_1^2 + F_1T_0T_1 + \alpha\frac{\partial}{\partial t}\left(T_0^2 + T_1^2 + T_0T_1\right)$$
$$I_2 = A_2 + B_2T_1 + C_2T_2 + D_2T_1^2 + E_2T_2^2 + F_2T_1T_2 + \beta\frac{\partial}{\partial t}\left(T_1^2 + T_2^2 + T_1T_2\right)$$
$$I_3 = A_3 + B_3T_2 + C_3T_3 + D_3T_2^2 + E_3T_3^2 + F_3T_2T_3 + \gamma\frac{\partial}{\partial t}\left(T_2^2 + T_3^2 + T_2T_3\right)$$
$$I_4 = A_4 + B_4T_3 + C_4T_4 + D_4T_3^2 + E_4T_4^2 + F_4T_3T_4 + \mu\frac{\partial}{\partial t}\left(T_3^2 + T_4^2 + T_3T_4\right)$$
$$I_5 = A_5 + B_5T_4 + C_5T_5 + D_5T_4^2 + E_5T_5^2 + F_5T_4T_5 + \eta\frac{\partial}{\partial t}\left(T_4^2 + T_5^2 + T_4T_5\right)$$

where A_i, B_i, D_i, E_i, F_i , $1 \le i \le 5$ and C_j, $2 \le j \le 5$ are all constants depending upon the value of physical and physiological parameters of dermal part, and

$A_1 = \frac{1}{2}hT_\infty^2$; $B_1 = LE - hT_\infty$; $D_1 = \frac{1}{2}\left(\frac{K^{(1)}}{l_1} + h\right)$; $E_1 = \frac{K^{(1)}}{2l_1}$; $F_1 = -\frac{K^{(1)}}{l_1}$

$A_2 = 0$; $B_2 = -P_1$; $C_2 = -2P_1$; $D_2 = \frac{1}{2R_1}$; $E_2 = \frac{1}{2R_1}$; $F_2 = -R_1$;

$A_3 = \frac{T_b^2 m (l_3 - l_2)^2}{4(l_4 - l_2)}$; $B_3 = -P_2(l_3-l_2)^2 - N_1(l_3^2+l_2l_3-2l_2^2-3l_1l_3+3l_1l_2)$;

$C_3 = P_2(4l_2l_3 - 2l_2^2 - 2l_3^2) + N_1(l_2l_3+l_2^2-2l_3^2-3l_1l_2+3l_1l_3)$;

$D_3 = \frac{1}{2}R_2 + \frac{1}{24}Q_1(l_3^3-3l_2l_3^2+3l_2^2l_3-l_2^3)$;

$E_3 = \frac{1}{2}R_2 + \frac{1}{8}Q_1(l_3-l_2)^3$;

$F_3 = -R_2 + \frac{1}{12}Q_1(l_3-l_2)^3$;

$A_4 = \frac{T_b^2 m}{4(l_4-l_2)}(l_4^2-l_3^2-2l_2l_4+2l_2l_3)$;

$B_4 = -P_2(l_4^2+l_3l_4-2l_3^2-3l_2l_4+3l_2l_3) - N_1(l_4^2+l_3l_4-2l_3^2-3l_1l_4+3l_1l_2)$;

$C_4 = P_2(l_3l_4+l_3^2-2l_4^2-3l_2l_3+3l_2l_4) + N_1(l_3l_4+l_3^2-2l_4^2-3l_1l_3+3l_1l_4)$;

$$D_4 = \frac{1}{2} R_3 + \frac{1}{24} Q_2(l_4^3+l_3l_4^2-5l_3^2l_4+3l_3^3-4l_2l_4^2-4l_2l_3^2+8l_2l_3l_4);$$

$$E_4 = \frac{1}{2} R_3 + \frac{1}{24} Q_2 (l_3^2l_4+l_3^3-5l_3l_4^2+3l_4^3-4l_2l_4^2-l_2l_3^2+8l_2l_3l_4);$$

$$F_4 = - R_3 - \frac{1}{12} Q_2 (l_3l_4^2+l_3^2l_4 - l_4^3 - l_3^3 - 4l_2l_3l_4+2l_2l_4^2+2l_2l_3^2);$$

$$A_5 = \frac{1}{2} mT_b^2 (l_5-l_4); \quad B_5 = - P_3 (l_5-l_4); \quad C_5 = P_3 (l_4-l_5);$$

$$D_5 = \frac{1}{2} R_4 + Q_3; \; E_5 = \frac{1}{2} R_4 + Q_3; \; F_5 = - R_4 + Q_3.$$

Where,

$$P_1 = \frac{s(l_2-l_1)^2}{6(l_4-l_1)}; \quad P_2 = \frac{T_b m}{6(l_4-l_2)}; \; P_3 = \frac{1}{2} (T_b m+s);$$

$$R_1 = \frac{K^{(2)}}{(l_2-l_1)}; \quad R_2 = \frac{K^{(3)}}{(l_3-l_2)}; \quad R_3 = \frac{K^{(4)}}{(l_4-l_3)}; \quad R_4 = \frac{K^{(5)}}{(l_5-l_4)};$$

$$Q_1 = \frac{m}{(l_3-l_2)(l_4-l_2)}; \quad Q_2 = \frac{m}{(l_4-l_3)(l_4-l_2)};$$

$$Q_3 = \frac{m(l_5-l_4)}{6}; \quad N_1 = \frac{s}{6(l_4-l_1)}.$$

$$\alpha = \frac{\rho c\, l_1}{2}; \quad \beta = \frac{\rho c\, (l_2-l_1)}{6}; \quad \gamma = \frac{\rho c\, (l_3-l_2)}{6}; \quad \mu = \frac{\rho c\, (l_4-l_3)}{6}; \eta = \frac{\rho c\, (l_5-l_4)}{6}$$

We differentiate I with regard to the nodal temperatures T_o, T_1, T_2, T_3, T_4 and T_5 and set $\frac{dI}{dT_j} = 0$, j = 0, 1, 2, 3, 4. Since $T_5 = T_b$ (the body core temperature), we get the system of equations in matrix form:

$$C\dot{T} + PT = W \qquad (3)$$

Where,

$$C = \begin{bmatrix} 2\alpha & \alpha & 0 & 0 & 0 \\ \alpha & 2(\alpha+\beta) & \beta & 0 & 0 \\ 0 & \beta & 2(\beta+\gamma) & \gamma & 0 \\ 0 & 0 & \gamma & 2(\gamma+\mu) & \mu \\ 0 & 0 & 0 & \mu & 2(\mu+\eta) \end{bmatrix}$$

$$T = \begin{bmatrix} T_o \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix},\ P = \begin{bmatrix} 2D_1 & F_1 & O & O & O \\ F_1 & 2(E_1+D_2) & F_2 & O & O \\ O & F_2 & 2(E_2+D_3) & F_3 & O \\ O & O & F_3 & 2(E_3+D_4) & F_4 \\ O & O & O & F_4 & 2(E_4+D_5) \end{bmatrix},\ \dot{T} = \begin{bmatrix} \frac{\partial T_0}{\partial T} \\ \frac{\partial T_1}{\partial t} \\ \frac{\partial T_2}{\partial t} \\ \frac{\partial T_3}{\partial t} \\ \frac{\partial T_4}{\partial t} \end{bmatrix}$$

NUMERICAL RESULTS AND DISCUSSION

Parameter values used in the model are as considered in Table 1 below.

Table 1: Parameter values considered [3, 4].

Parameter	Value	Unit
$K^{(1)}$	0.20934	w/m^{0}C
$K^{(2)}$	0.20934	w/m^{0}C
$K^{(3)}$	0.031401	w/m^{0}C
$K^{(4)}$	0.031401	w/m^{0}C
$K^{(5)}$	0.41868	w/m^{0}C
L	2.4×10^6	J/kg
h	6.2802	w/m$^{2\ 0}$C
E	1.9×10^{-4}	w/m^2
ρ	1050	kg/m^2
C	3475.044	J/kg
M	2198.07	w ^{0}C
S	1256.04	w/m^3

Table 2: Two sets of dermal layers

Sets	l_1(m)	l_2(m)	l_3(m)	l_4(m)	l_5(m)
I	0.0005	0.001	0.002	0.0035	0.005
II	0.0005	0.001	0.0025	0.004	0.009

And, for nodal temperatures at t = 0, we are taking the equation T(x, t) = T(x, 0) + px considering initial temperature 22.87^{0}C at skin surface because at normal atmospheric temperatures skin surface temperature is around 22.87^{0}C. To solve the system of ordinary differential equation (3), we use Crank-Nicolson method. According to the method, the system of equation (3) can be written as

$$\left(C + \frac{\Delta t}{2} P\right) T^{(i+1)} = \left(C - \frac{\Delta t}{2} P\right) T^{(i)} + \Delta t W \qquad (4)$$

where Δt is the time interval.

Using the above numerical values in equation (4) the profiles for temperature distribution in the layers of dermal part so obtained is as shown in figures below taking various atmospheric temperatures.

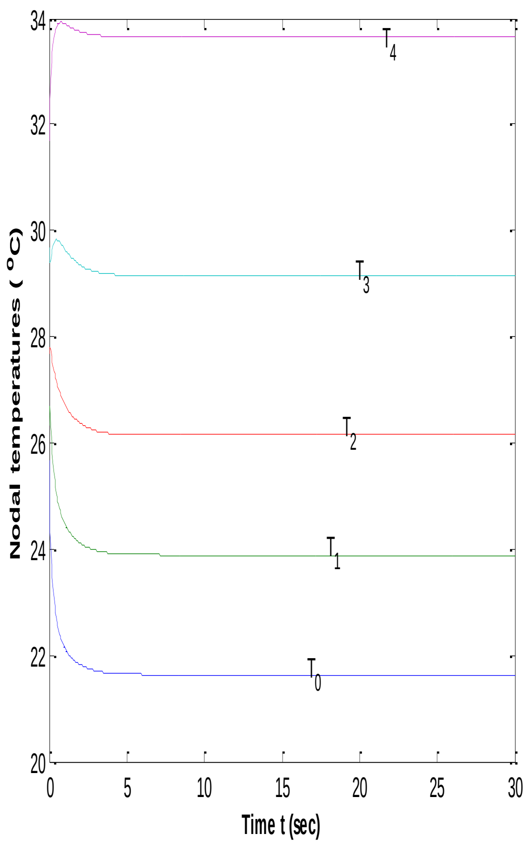

Fig.1: Nodal Temperature for Set-I at $T_a = 45^0$C

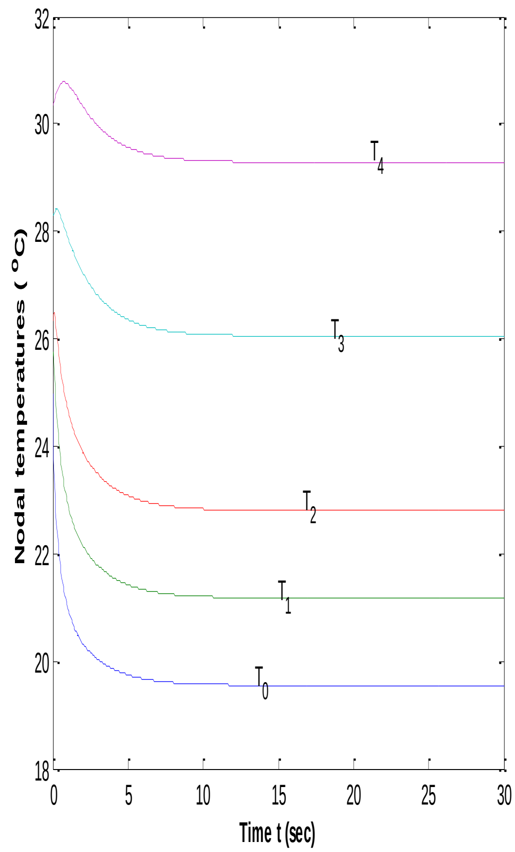

Fig. 2: Nodal Temperature for Set-II at $T_a = 45^0$C

The temperature distribution profiles for nodal temperatures T_i (i=0, 1, 2, 3, 4) for the two set of skin thicknesses at different atmospheric temperatures with time dependence are shown in figures1-6. From these figures it has been observed that the curve for T_i rise faster in set I and reach steady state case earlier than set II. It is also observed that steady state temperatures at these depths are higher for set I than to set II. These are due to lower thickness of set I than to set II. These figures reveal that the temperatures Ti at high atmospheric temperatures are higher than lower atmospheric temperatures. This is due to the more effect on body temperature at higher atmospheric temperatures. So the thicknesses of dermal layers and different atmospheric temperatures have the significant effects in temperature profiles.

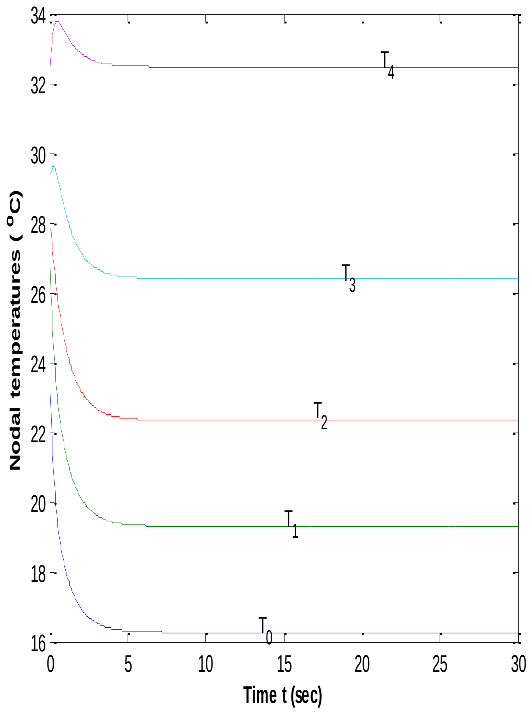

Fig. 3: Nodal Temperature for Set-I at $T_a = 37^0$C

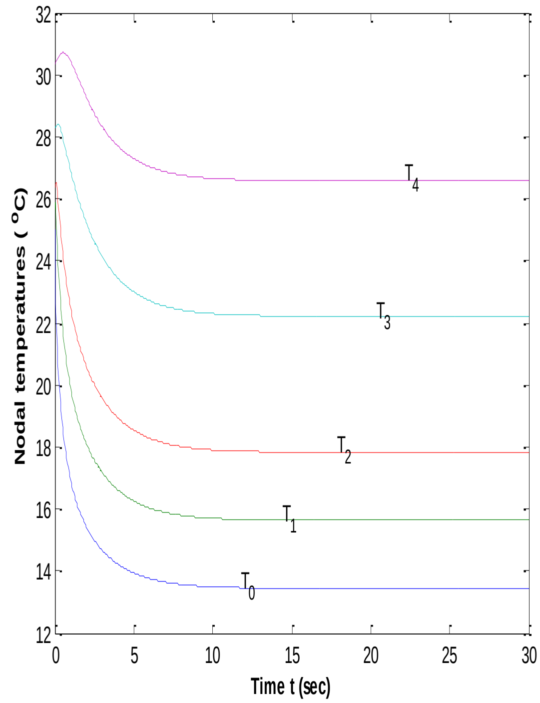

Fig. 4: Nodal Temperature for Set-II at $T_a = 37^0$C

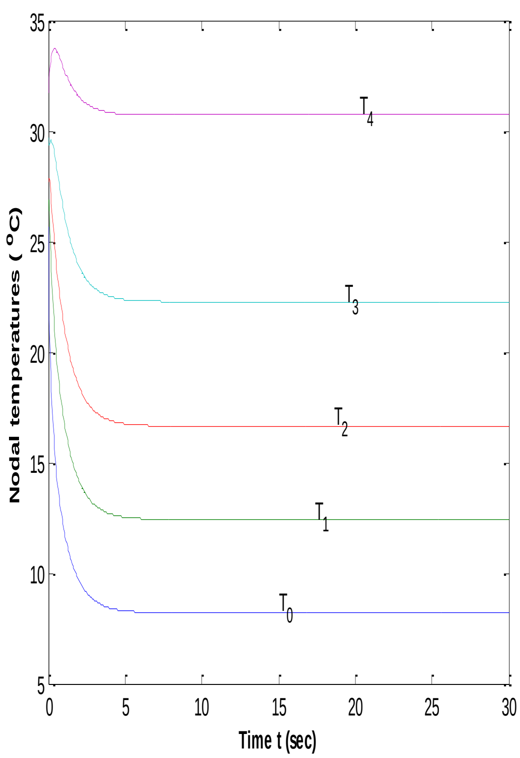

Fig. 5: Nodal Temperature for Set-I at $T_a = 25^0$C

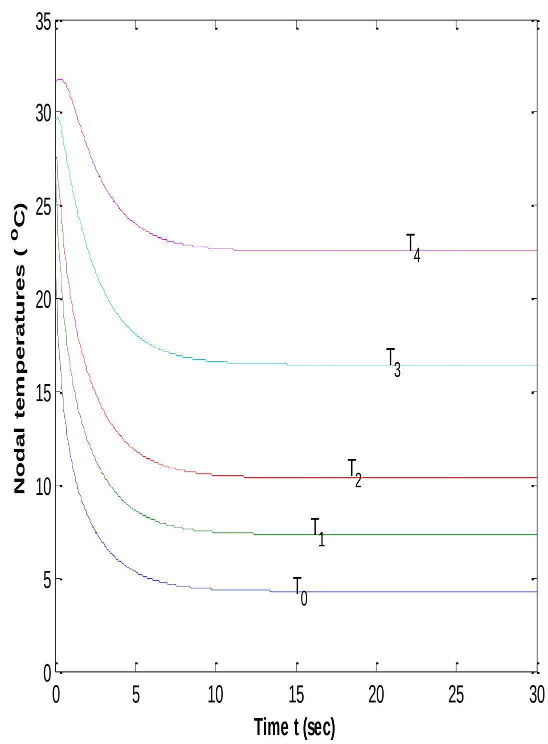

Fig. 6: Nodal Temperature for Set-II at $T_a = 25^0$C

CONCLUSION

The results are comparable with those obtained in [4] who considered only three layers of skin epidermis, dermis and subcutaneous tissue. The differences in the results whatsoever may be, due to the extension up to five layers of dermal part – Stratum Corneum, Stratum Germinativum, Papillary Region, Reticular Region and Subcutaneous Tissue. Our model gives better profiles for temperature distribution in dermal layers. This is because the model has incorporated more feasible layers and has taken significant biophysical parameters.

The epidermis layer contains five layers and dermis contains two layers. So biologically dermal region has eight layers. We may further extended the model considering the eight layers as eight elements in one dimensional case. It may be extended for two dimensions and three dimensions. The study may be carried out for predication of thermal response for humans exposed to hot or cold environments. The model may be extended by considering the field variable as quadratic or of higher order polynomial in FEM technique to get better temperature profiles.

The different temperature profile can be viewed by considering this model at clinical situation, indoor climate and risk under stressful condition. It is helpful for the medical scientist to suggest and evaluate for new treatment strategies. It is also helpful to maintain indoor temperature. The presented model can be extended to explore several problems mentioned above.

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