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MOBILITY ANALYSIS OF KINEMATIC CHAINS

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ABSTRACT

In the present work a simple and efficient method is proposed to identify whether a kinematic chain posses total, partial or fractionated mobility. The proposed method uses the chain flow values (CFV) derived from the flow matrix of the given kinematic chain and successfully applied to all known cases of 2 and 3 degree of freedom planar kinematic chains. Since the method is systematic and efficient, it can be applied to the more complex chains which not have been reported in the literature yet. This study will be helpful in dividing the frame and input links from the view point of mobility. Some examples are provided to demonstrate the effectiveness of this method.

Keywords: Degree of freedom (DOF), Contour, Chain flow value (CFV).

INTRODUCTION

The determination of the type of degrees of freedom in kinematic chains is one of the most important and challenging problem in the structural analysis of kinematic chains. Several vigorous studies have been reported in literature concerning the type of degree of freedom in kinematic chains. Davies[3] presented his definitive work with some very useful theorems on the type of degree of freedom in kinematic chains based on the graph theory. Later Sen and Mruthyunjaya[14] presented several counter examples to Davies' theorems. Mruthyunjaya and Raghavan[11] used the Davies' theorems and obtained algebraic procedures based on a link-link adjacency matrix representation of kinematic chains for the detection of fractionated and partial mobility of chains. Mruthyunjaya and Raghavan[12] also presented computer implementations for detecting the type of mobility and for deriving distinct mechanisms and driving mechanisms, from a kinematic chain. Agrawal and Rao[1] proposed methods for detection of fractionated mobility using the path loop connectivity matrix. They presented the properties of the loop freedom matrix for fractionated chains. In a subsequent paper Agrawal and Rao[2], presented a method for the analysis of the mobility properties of a kinematic chain by its loop freedom matrix and its permanent function. Harary and Yan[5] gave a precise definition for a kinematic chain in terms of hyper graphs satisfying certain axioms. Liu and Yu[7] presented a procedure based on the information obtained by calculating the basic loops and their DOF for identifying and classifying multi-DOF and multiple loop mechanisms. Rao and Pathapati[13] presented a loop based detection of isomorphism of chains. Their method is reported to give the information on mobility without extra computational effort. Tischler et al.[15] formulated the concepts of total, partial and fractionated mobility in terms of the variety of a chain. Lee and Yoon[6] developed an algorithm for identifying the mobility type of a planar mechanism that is valid even when the graph of the mechanism is non-planar. While

Lee and Yoon provide a working approach, the underlying mathematical reasoning for the algorithm's validity was not published in their work limiting the credibility of the algorithm. In this study, a new procedure is presented to determine the type of degree of freedom using the concept of chain flow value. The procedure is systematic and applicable to both 2 and 3 DOF planar kinematic chains.

DEFINITION OF TERMINOLOGY

In order to clarify the terminologies used in this paper they are defined as follows:

(i) Mobility – This has the same meaning as the degrees of freedom of mechanisms and is used interchangeably.

(ii) Planar kinematic chain - Kinematic chain which can be drawn on a plane without any crossed links.

(iii) Contour: A counter is defined as a closed loop allowing to pass through kinematic joints belonging to it.

(iv) Internal contour: It is defined as a closed loop allowing one to travel through kinematic joints belonging to the interior of a kinematic chain. They are labelled as (1), (2), (3) etc. These contours for any chain can easily be identified by visual inspection and do not depend upon the manner in which the chain is drawn.

(v) Sub contour: They are the combinations of the inter contours For example, if the internal contours are (1), (2) and (3), the sub contours for that chain are (1-2), (1-3), (2-3) and (1-2-3), to avoiding repetition of a combination similarly if the chain has 4, internal contours all possible sub contours for the chain are (1-2), (1-3), (1-4), (2-3), (2-4), (3-4), (1-2-3), (1-2-4), (1-3-4), (2-3-4) and (1-2-3-4). The last sub counter for any chain includes all the counters and is also known as *external contour*.

(vi) Link Flow Matrix**:** For an n-link contour it is defined as an n x n square matrix whose any i^{th} and j^{th} element F_{ij} is defined as

$[FM] = \{F_{ij}\}$ n x n ; Where

F_{ij} {= Minimum number of joints between link i and j.

= 0 if i is equal to j}

Off course all the diagonal elements F_{ij} =0

Thus the form [FM] matrix will be

$$[FM] = \begin{bmatrix} 0 & F_{12} & F_{13} & - & F_{1n} \\ F_{21} & 0 & F_{23} & - & F_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ F_{n1} & F_{n2} & F_{n3} & - & 0 \end{bmatrix}$$

(vii) Chain Flow Value [CFV]: It is the algebraic sum of all elements of link flow matrix.

DEGREES OF FREEDOM

The type of freedom in multi degree of freedom chain is an important consideration in order to select the frame and actuator (input) links from the view point of mobility. A multi degree of freedom kinematic chain possesses one of the following types of freedom.

(i) Full degree of freedom: A chain is said to have a Full degree of freedom F if any F links of the chain can be moved independent of one another, relative to any other link and the motion of all the remaining links is dependent on the motion of all these F and not less than F links. In other words a kinematic chain is said to have total degree of freedom F if every contour has mobility larger than or equal to F. Thus, a multi (two) DOF kinematic chain has Full degree of freedom if there is an absence of one DOF four bar contour in that particular chain. This type of freedom gives total freedom of selection of any link of given multi degree of freedom kinematic chain for input. For example, if two DOF kinematic chain has Full degree of freedom then any two links of this chain can be selected as input links.

(ii) Partial degree of freedom: A chain which does not satisfy the conditions of Full degree of freedom is said to have partial degree of freedom. This type of freedom gives partial freedom of selection of links of given multi degree of freedom kinematic chain for input. For example, if two DOF kinematic chain has Partial degree of freedom then any two links of this chain can not be selected as input links.

(iii) Fractionated degree of freedom: The concept of Fractionated degree of freedom was first introduced by Monolescu[8]. It is a special case of Partial degree of freedom in which a chain also consists of a link (called the separation link) by cutting which into two, the chain of F degree of freedom can be split into two separate chains of DOF F_1 and F_2 such that $F_1 + F_2 = F$. Thus, a kinematic chain has Fractionated degree of freedom if it contains at least one link which must have at least four pairs.

IDENTIFICATION OF TYPE OF FREEDOM

Using the various structural properties of different types of DOF as explained above a simple method is proposed to identify whether a chain posses a particular type of freedom. The method utilised the fact that a four bar contour has single DOF and a five bar contour has two DOF and so on. The CFV of smallest contour including sub contours of a chain will decide the type of freedom.

This method includes three steps of identification of type of freedom
1. Determination of CFV of a four bar contour one degree of freedom chain.
2. Identification by observation the presence of a four bar contour in a given kinematic chain.
3. Confirm presence of four bar contour in a given kinematic chain by comparing contour's CFV derived from given kinematic chain flow path matrix with CFV of the four bar contour one degree of freedom chain derived in step 1.

Step 1 – Determining CFV of a four bar contour one DOF chain as shown in Figure - 1.

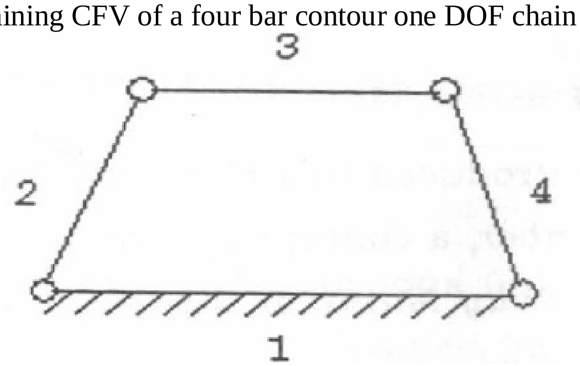


Figure 1- Four bar one degree of freedom chain

Link flow matrix

$$
\begin{array}{c c}
\text{Link} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{pmatrix}
\end{array}
$$

So the CFV of a four bar one DOF contour is 16.

Identification of full degree of freedom

Given kinematic chain of multi DOF may have full degree of freedom i.e. freedom of selection of any links as input link of given kinematic chain, if and only if either there is an absence of four bar contour in the chain that is ensured by comparing the CFV of all the contour of given kinematic chain with the CFV of a four bar contour one DOF chain or there is an presence of at least one link that have connectivity higher than three**.**

Identification of partial degree of freedom

Given kinematic chain of multi DOF may has partial degree of freedom i.e. partial freedom of selection of any links as input link of given kinematic chain, if and only if there is a presence of a contour that have CFV equal to CFV of a four bar contour chain of one DOF ensuring an presence of four bar contour in the chain.

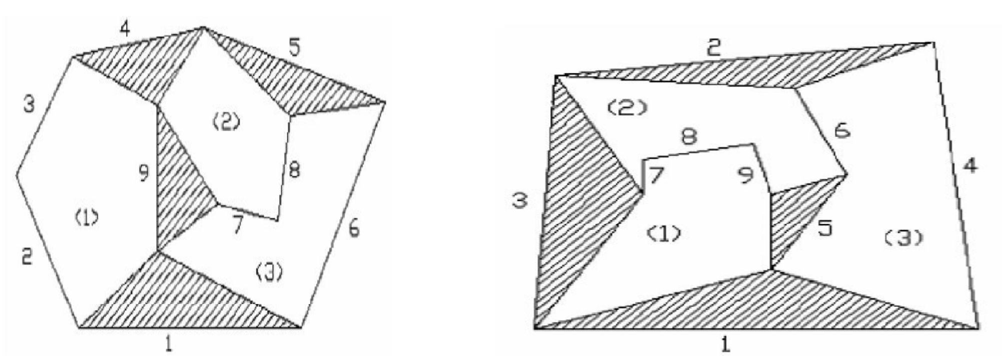
Identification of fractionated degree of freedom

In a given kinematic chain Fractionated degree of freedom will exist if and only if there is not only the presence of a four bar contour but also the presence of at least one link that is common to more than 50% of total contours of the chain .

Although the above procedure is explained for two degree of freedom chains, it holds good for three degree of freedom chains also. For the chains to have partial freedom, the contours should have two degree of freedom or five bars.

APPLICATION TO KINEMATIC CHAINS

Example 1: Consider nine links two degree of freedom chain as shown in Figure – 2 (a) and (b).

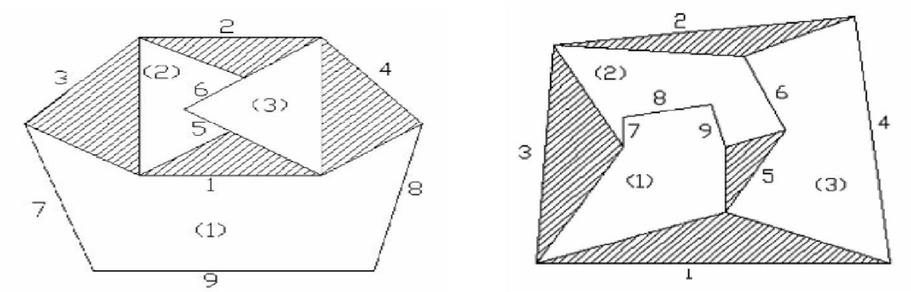


(a) (b)

Figure - 2. Nine links two degree of freedom chains with full freedom

Step 2 – The chains as shown in Fig. 2(a) and (b), both have three internal contours (1), (2) and (3) and all possible sub contours are (1-2), (1-3), (2-3) and (1-2-3).
Here none of the contour has four bars. Thus there is an absence of four bar contour of one degree of freedom in the given nine links two degree of freedom kinematic chains, so both the chains have Full degree of freedom.

Example 2: Consider nine links two degree of freedom chain as shown in Figure 3 (a) and (b).



(a) (b)

Fig. 3 - Nine links two degree of freedom chains with partial freedom

Step 2 – The chains as shown in Fig. 3 (a) and (b), both have three internal contours (1), (2) and (3) and all possible sub contours are (1-2), (1-3), (2-3) and (1-2-3).
Here although none of the internal contour have four bar but the sub contour (2-3) of chain 3 (a) and external contour (1-2-3) of chain 3 (b) are four bar contours.
Step 3 – Confirmation of presence of four bar contour
Flow path matrix for sub contour (2-3), between links 1, 2, 3, 4, of chain 3 (a)

$$\begin{array}{c c} \text{Link} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

So the CFV of sub contour (2-3) is 16.

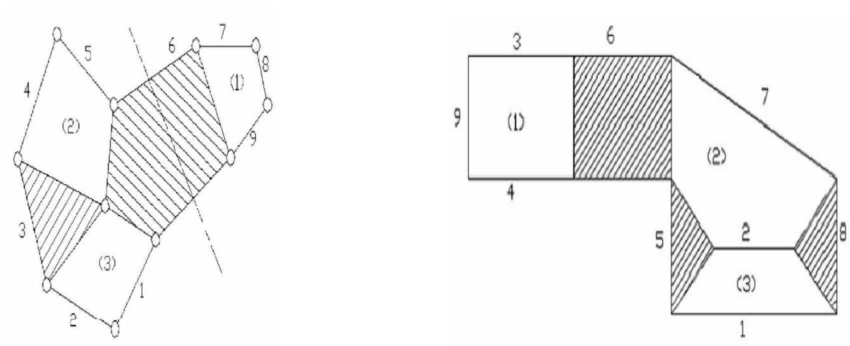
Flow path matrix for external contour (1-2-3), between links 1, 2, 3, 4, of chain 3 (b)

$$\begin{array}{c c} \text{Link} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

So the CFV of contour (1-2-3) is also16.

The CFV of these matrices are similar to CFV of a four bar contour one DOF chain as determined in Step -1. This gives the confirmation of presence of four bar contour one DOF chain in given nine links two degree of freedom kinematic chains, thus both the chains have partial degree of freedom.

Example 3: Consider nine links two degree of freedom chain as shown in Figure- 4 (a) and (b).



(a) (b)

Fig. 4 - Nine links two degree of freedom chains with fractionated freedom

Step 2 – The chains as shown in Fig. 4 (a) and (b) have three internal contours (1), (2) and (3) and all possible sub contours are (1-2), (1-3), (2-3) and (1-2-3).

Here for chain 4 (a) all three internal contours have four bars and link no. 6 is sharing all the contours and for chain 4 (b) internal contours (1) and (3) have four bars and link no. 6 is sharing internal contour (1), (2) and sub contour sub contours (1-2), (2-3) and (1-2-3), more than 50% of the total contours.

Step 3 – Confirmation of presence of four bar contour

Flow path matrix for internal contour (2), between links 3, 4, 5, 6, of chain 3 (a)

$$\begin{array}{c c}
\text{Link} & \begin{matrix} 3 & 4 & 5 & 6 \end{matrix} \\
\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}
\end{array}$$

CFV is equal to 16.

The CFV of this matrix is similar to CFV of a four bar contour one DOF chain as determined in Step -1. This gives the confirmation of presence of four bar contour one DOF chain in given nine links two degree of freedom kinematic chains, and link no. 6 of both the chains is sharing contours more than 50% of the total contours. Thus both the chains have fractionated degree of freedom.

RESULTS

The proposed method is applied to two and three degree of freedom planar kinematic chains, up to 10 links to identify their type of freedom. The result of 7 links, 2 DOF kinematic chains is in agreement with the result reported by Davies[4] and the results of 9 link, 2 DOF kinematic chains is in agreement with the result reported by Mruthyunjaya[9]. The results of 10 link 3 DOF kinematic chains is in agreement with the result reported by Mruthyunjaya[10] in the number of total DOF kinemaic chains. However, the number of partial DOF kinemaic chains is higher.

CONCLUSIONS

Though, on proof being offered, the author's strongly believe that this method is able to identify the type of freedom of two and three degree of freedom planar kinematic chains, up to 10 links. The main advantage of this method over other previous methods is that this method is extremely simple in its process and of time savvy nature with perfect reliability from the point of view of results. A further study is required for analysis of non planar kinematic chains.

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