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FOUNDATIONS OF GEOMETRY – IV

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Gerolamo Saccheri (1667 – 1733), an Italian geometer proved many theorems of absolute geometry. Initially, it was not his goal to prove these theorems. His goal, like his predecessors of earlier centuries, was to prove Euclid's fifth (or parallel) postulate as a theorem. He tried to prove the parallel postulate by assuming that it is false and hence come to a contradiction. His assumption was that given any line l and any point P not on l, then at least two different lines through P do not meet l. During his attempt to prove this "false theorem" he proved great many theorems on the basis of this assumption that he would reach a contradiction. In the end he was discouraged and asked his reader to accept one of his theorems is contrary to the nature of straight line and is false. Hence the assumption stated above is false.

But what Saccheri failed to notice is that not his "false theorem" is actually false but it is a theorem which is true in a new kind of geometry. Saccheri too, like other geometers, with his absolute faith on the geometry of Euclid could not think of the existence of any other geometries except Euclidean and thus missed the chance of being credited as the founder of a new geometry. The geometry where the assumption of Saccheri is true is now known as the "Hyperbolic geometry" and the credit for the discovery of this geometry is shared by the Hungarian Johann Bolyai and the Russian Nicolai Lobachevsky.

Janos Bolyai (in German Johann Bolyai, 1802 – 1860) was the son of a prominent Hungarian mathematician Wolfgang Bolyai and was an army officer by profession. It is said that the history of mathematical research in Hungary began with Wolfgang Bolyai. Johann learned mathematics at an early age from his father and soon took a special interest in trying to prove Euclid's parallel postulate. While doing so he developed a whole new geometry while he was a student in Vienna in 1823. Johann was so excited about his new discovery that he put it as creating a whole new universe out of nothing. He discussed his new ideas with his father and his father included Johann's work in an appendix to his own large geometric treatise in 1832 and sent a copy of his treatise to one of his old friend and a great mathematician Karl F. Gauss. Gauss was universally acknowledged as one of the greatest mathematicians of his time.

But Gauss' reply was something of a shock to Johann from which he could never recover. Gauss said that he could not praise the discovery of Young Johann because if he did so then it would be praise to himself for he himself has discovered similar results and his mind had been partly occupied for the last thirty five years by this new geometry. But he did not publish the results due to the fear that there would be very few who can understand these new ideas. To say something against Eudid's geometry would be certainly unpopular and Gauss did not like to tarnish his image by linking his name to an idea which would bring criticism from others.

Another great mind who also discovered similar new ideas about geometry was Nicolai Lobachevsky (1793 – 1856) of Russia. Nicolai spent his entire academic life at the University of Kazan, first as a student, then an assistant, and finally as a full Professor. He thought of the possibility of proving Euclid's parallel postulate several years earlier than he began thinking about a possible new geometry in 1823. He first delivered a lecture about this geometry in the university in 1826 and published a paper on it in 1829. He devoted his entire life to this work and continued publishing his works until his death in 1856. A book also appeared in 1840 in German. At the University Labachevsky would not only give lectures in pure mathematics but in Astronomy and theoretical physics too. In 1827 he was elected Rector of the University and continued holding the post for 19 years. Despite his heavy duty as a Rector, he developed his new geometry in such a detail that it is called "Lobachevsky Geometry".

Bolyai and Lobachevsky made their revolutionary discoveries about the same new geometry within a year or two of each other. But each of them never new of other's existence. Lobachevsky, during his entire life time, never heard of Bolyai or his discoveries. However, Bolyai in 1848 learned of Lobachevsky's work through a copy of the book which the latter published in 1840. It is one of the remarkable coincidences of science that Bolyai and Lobachevsky's discoveries of new geometry are comparable to the simultaneous invention of calculus by Newton and Leibniz. Despite his father's urge to Johann to publish his new discoveries, his works remained unpublished and thus the credit for discovering the new geometry went to Nicolai Lobachevsky.

Lobachevsky called Euclidean geometry 'practical' and his own geometry 'imaginary'. But imaginary does not mean that Lobachevsky geometry was just a pure logical system (which in modern time we call it a complete system) Lobachevsky used his imaginary geometry to compute more than two hundred integrals. At present it is known that there is a profound connection between this geometry and many other branches of mathematics. But Bolyai and Lobachevsky, both of them, failed to get recognition for the importance of their works. But only after the death of Lobachevsky did the scientific world came to realize the importance of his researches. Before Lobachevsky only the Euclidean geometry provided a conceivable notion about space. But with Lobachevsky's imaginary geometry or the non-Euclidean geometry the view point of space has been broadened with far reaching consequences. It has laid the foundation of modern notion of an abstract space and established geometry as a science. Also, there are other applications in mathematics and related subjects.

In spite of the efforts of geometers to prove the fifth or parallel postulate over a period of more than twenty centuries, the problem remained roughly at the same stage as it was in the time of Euclid. But the first decade of the nineteenth century saw this problem solved. But the mathematics community continued considering Euclidean geometry more or less as complete and was not prepared to receive these new ideas more enthusiastically.

We mentioned, earlier, that many of the propositions obtained by Lobachevsky were obtained earlier by Saccheri and Lambert in connection with the acute angle hypothesis. Saccheri used to the acute angle hypothesis to derive a contradiction of this logical system of new geometry which he failed but Lobachevsky thought that the propositions thus obtained by the acute angle hypothesis forms a new geometry. Thus the preliminary results obtained by Saccheri and Lobachevsky are the same but Lobachevsky established

that the propositions and theorems obtained were a form of new geometry (called imaginary geometry by Lobachevsky). This imaginary geometry is also, like Euclidean geometry, is free from any logical contradiction. However, Lobachevsky realized that if there were any contradictions, it was difficult to know beforehand at which stage of development they might be discovered. So, in order to make his geometry free from any contradictions, Lobachevsky had recourse to a deep algebraic analysis of its equations and solved the problem as far as possible at this time. The results of Lobachevsky's investigations can be summarized briefly as follows:

(a) The parallel postulate is not a necessary consequence of the other postulates of geometry (in other words, the parallel postulate is logically independent of other postulates).

(b) The fifth postulate does not follow from other postulates because this postulate remains true in Euclidean geometry but there is another form of geometry where this postulate is not true.

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This article is the fourth installment of a series article by the author