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A NEW METHOD TO DETECT ISOMORPHISM IN KINEMATIC CHAINS

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ABSTRACT
The present work deals with the problem of detection of isomorphism, which is frequently encountered in structural synthesis of kinematic chains (KC). A new method based on theoretic approach, easy to compute and reliable is suggested in this paper. The KC are represented in the form of the Joint-Joint [JJ] matrices. Two structural invariants derived from the characteristic polynomials of the [JJ] matrix of the KC are the sum of absolute characteristic polynomial coefficients (shortened as **∑JJ**) and maximum absolute value of the characteristic polynomial coefficient (shortened as **MJJ**). These invariants are used as the composite identification number of a KC and mechanisms. It is capable of detecting isomorphism in all types of planer kinematic chains. This study will help the designer to select the best KC and mechanisms to perform the specified task and avoid duplication at the conceptual stage of design. The application this study is in research and development industries. The proposed method has been explained with the help of examples. The KC of 1-F, 8-links, and 10-links as well as 2-F, 9-link has been tested and the results are in complete agreement with the available literature of structural synthesis.

KEY WORDS: KC; Isomorphism; [JJ] matrix; Characteristic polynomial; ∑JJ; MJJ.

INTRODUCTION
In a mechanism design problem, systematic steps are type synthesis, structural/number synthesis and dimensional synthesis. Structural synthesis of the KC and mechanism has been the subject of a number of studies in recent years. One important aspect of structural synthesis is to develop the all-possible arrangements of KC and their derived mechanisms for a given number of links, joints and degree of freedom, so that the designer has the liberty to select the best or optimum mechanisms according to his requirements. In the course of development of KC and mechanisms, duplication may be possible. For this reason, many methods have been proposed by many researchers to check for duplication or in other words, to detect the isomorphism among the KC. Most of these methods are based on the adjacency matrix (Uicker, 1975) and the distance matrix (Rao, 1988). For determining the structurally DM of a KC, the link disposition method (Mruthuynjaya, 1984), the flow matrix method (Nageshwara Rao, 1996) and the row sum of extended distance matrix methods (Aas Mohammad, 1999) are used. Minimum code (Ambekar, 1987), characteristic polynomial of matrix (Mruthuynjaya, 1979), identification code (Ambekar, 1985), link path code (Agrawal, 1996), summation polynomial (Shende, 1994) are used to characterize the KC. With regard to these methods, either there is a lack of uniqueness or they take too much time.

OBJECTIVES

In this paper, the authors' objective is to develop a new, reliable, and efficient method to detect isomorphism in KC and mechanisms. It will help the designer to select the best KC and mechanism to perform the desired task, at the conceptual stage of design. The proposed method is presented by comparing the structural invariants ∑JJ and MJJ of [JJ] matrices. These invariants may also be used not only to detect isomorphism in the KC and mechanisms having simple joints but also the KC having Co-spectral graph. The method is explained with the help of examples of planner KC having all simple joints.

Definitions of terminology

A number of new terms have been developed in the present work for a KC. They are defined in the following paragraph. The joints of a chain are assigned positive integers 1, 2, 3…, n as their names while the small English letters a, b, c, etc. are used for labeling the links of a KC. It may be mentioned here that links and joints of a chain are arbitrarily labeled.

Degree of link d (li): The degree of link actually represents the type of link like binary, ternary, quaternary etc. Let the degree of i^{th} link in a KC be designed as d (l_i).

d (l_i) =2, for binary link, d (l_i) =3, for ternary link,
d (l_i) = 4, for quarter nary link, d (l_i) =k, for k-nary link.

Matrix representation of the kinematic chain: The (0, 1) adjacency matrix and the distance matrix are generally used to represent the kinematic graph of a KC. The adjacency matrix shows only the connectivity between adjacent vertices/links. The distance matrix has also the relation between the links that are not directly connected to each other in the form of shortest path distance. However, both adjacency and distance matrices are not able to furnish the information about the types of links those are directly connected with a joint or with the shortest path distances respectively. A generalized matrix representation is made in literature [7] in which the elements of adjacency matrix a_{ij} represents the multiplicity (type) of the joint The value of a_{ij} is 1 if the joint between i^{th} and j^{th} link is a simple joint, 2 if it is double joint, 3 if it is ternary joint and so on. In the present paper the [JJ] matrix is used which is based upon the connectivity of the joints through a link.

The joint-joint [jj] matrix: This matrix is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size n x n, where n is the number of joints in a KC.

$$[JJ] \quad \left\{ L \right\}_{n \times n} \qquad \text{------------------- (1)}$$

Where

$$L_{ij} \begin{cases} = \text{Degree of link between } i^{th} \text{ and } j^{th} \text{ joints those are directly connected} \\ =0, \text{ if joint i is not directly connected to joint j} \end{cases}$$

Off course all the diagonal elements $L_{ii} = 0$

Thus the form of [JJ] matrix will be:

$$[JJ] = \begin{pmatrix} 0 & L_{12} & L_{13} & - & - & - & L_{1n} \\ L_{21} & 0 & L_{23} & - & - & - & L_{2n} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - \end{pmatrix}$$

Characteristic polynomial of [jj] matrix: The characteristic polynomial [1] is generally derived from (0, 1) adjacency matrix. The roots of n^{th} order characteristic polynomial are the set of n-eigen values called as Eigen spectrum. Many researchers have reported co-spectral graphs (the non-isomorphic graphs having same Eigen spectrum derived from (0, 1) adjacency matrix). The Proposed [JJ] matrix has additional information about the types of links existing in a KC. Therefore, it is expected that the characteristic polynomial and its coefficients will be unique to clearly identify the KC and even KC with co-spectral graphs. D (λ) gives the characteristic polynomial of [JJ] matrix. The monic polynomial of degree n is given by equation (2).

$$| (JJ - \lambda I) | \quad = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \text{---------} \; a_{n-1}\lambda + a_n \quad \text{--------------- (2)}$$

Where; n = number of simple joints in KC and
1, a_1, a_2, a_{n-1}, a_n are the characteristic polynomial coefficients.
The two important properties of the characteristic polynomials are

1. The sum of the absolute value of the characteristic polynomial coefficients (∑JJ) is an invariant for a [JJ] matrix. i.e.
   $| 1 | + \; | a_1 | + | a_2 | + \text{-----} + | a_{n-1} | + | a_n | = \text{invariant}$
2. The maximum absolute value of the characteristic polynomial coefficient (MJJ) is another invariant for a [JJ] matrix.

Structural Invariants '∑JJ' and 'MJJ': The values of characteristic polynomial coefficients are invariants for a [JJ] matrix. To make these [JJ] matrix characteristic polynomial coefficients as a powerful single number characteristic index, new composite invariants proposed. These indices are **'∑JJ'** and **'MJJ'**. These invariants are unique for a [JJ] matrix and may be used as identification numbers to detect the isomorphism among simple jointed KC and mechanisms. The characteristic polynomial coefficient values are the characteristic invariants for the chains and mechanisms. Many investigators have reported co-spectral graph (non-isomorphic graph having same Eigen spectrum). But these Eigen spectra (Eigen values or characteristic polynomial coefficient) have been determined from (0, 1) adjacency matrices. The proposed [JJ] matrix provides distinct set of characteristic polynomial coefficients of the KC having co-spectral graph. Therefore, it is hoped that the structural invariants **'∑JJ' and 'MJJ'** are capable of characterizing all KC and mechanisms uniquely. Hence, it is possible to detect isomorphism among all the given KC.

Isomorphism of Kinematic Chains:

Theorem: Two similar square symmetric matrices have the same characteristic polynomial. [11].

Proof: Let the two KC are represented by the two similar matrices A and B such that $B = P^{-1} AP$, taking into account that the matrix λI commutes with the matrix P and $| P^{-1} | = | P |^{-1}$. Since the determinant of the product of two square matrices equals the product of their determinants, we have

$$
\begin{aligned}
| B - \lambda I | &= | P^{-1} A P - \lambda I | \\
&= | P^{-1}(A - \lambda I) P | \\
&= | P^{-1} | \, | (A - \lambda I) | \, | P | = | A - \lambda I |
\end{aligned}
$$

Hence, D (λ) of 'A' matrix = D (λ) of 'B' matrix.

D (λ) =characteristic polynomial of the matrix.

It means that if D (λ) of two [JJ] matrices representing two KC is same, their structural invariants '∑JJ' and 'MJJ' will also be same and the two KC are isomorphic otherwise non-isomorphic chains.

Proposed test-basis: The kinematic chains are complex chains of combination of binary, ternary, and other higher order links. These links are joined together by simple pin joints. It is the assembly of link/pair combination to form one or more closed circuits. While considering structural equivalence it is essential to consider type of links/joints and layout of the links in the assembly. An identification number is assigned to links. Thus, a binary link has a value of two, ternary three, quarter nary four and so on. Link values are used to assigned values to [JJ] matrix and these are utilized to identify the layout of the KC.For detecting isomorphism in KC, a [JJ] matrix is determined and composite structural invariants [∑JJ] and [MJJ] of [JJ] matrix are determined using software **MAT LAB** . If the structural invariants '∑JJ' and 'MJJ'of the two KC are same then both the KC are considered as isomorphic otherwise non-isomorphic.

***Illustrative example* – 1**

The first example concerns two KC with 12 bars, 16 joints, single-degree of freedom as shown in Fig.1. The task is to examine whether these two KC are isomorphic.

[JJ] Matrix of KC shown in Fig.1 (a)

[JJ] Matrix of KC shown in Fig.1 (a) using equation (1) is given by equation (3)

$$[JJ]=\begin{pmatrix}
0&3&3&2&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&3&0&0&0&0&2&0&0&0&0&0&0&0&0\\
3&3&0&0&0&0&0&0&0&0&0&0&0&0&0&2\\
2&0&0&0&3&3&0&0&0&0&0&0&0&0&0&0\\
0&0&0&3&0&3&0&0&0&0&0&0&0&3&3&0\\
0&0&0&3&3&0&3&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&3&0&3&3&3&0&0&0&0&0&0\\
0&2&0&0&0&0&3&0&3&0&0&0&0&0&0&0\\
0&0&0&0&0&0&3&3&0&0&0&3&3&0&0&0\\
0&0&0&0&0&3&3&0&0&0&3&3&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&3&0&2&0&0\\
0&0&0&0&0&0&0&0&3&3&3&0&3&0&0&0\\
0&0&0&0&0&0&0&0&3&0&0&3&0&0&3&3\\
0&0&0&0&3&0&0&0&0&0&2&0&0&0&3&0\\
0&0&0&0&3&0&0&0&0&0&0&0&3&3&0&3\\
0&0&2&0&0&0&0&0&0&0&0&0&3&0&3&0
\end{pmatrix} \quad \text{------------------ (3)}$$

The characteristic polynomial coefficients for KC shown in Fig.1 (a)

The characteristic polynomial coefficients derived form [JJ] matrix of KC shown in Fig.1(a) using MAT LAB software are :
0.0000, 0.0000, -0.0000, -0.0000 0.0000 , 0.0000 , -0.0001, -0.0004 , 0.0016 , 0.0109 , -0.0056, -0.1239 , -0.1243, 0.4736, 0.7954 , -0.4396 , -1.0219

Structural Invariant for KC Shown In Fig.1 (a)

The set of structural invariant for KC Shown in Fig.1 (a), derived from [JJ] matrix using software MAT LAB are:
[∑JJ] = 2.9971e+010 , [MJJ] =1.0219e+010

Similarly

[JJ] Matrix of KC shown in Fig.1 (b)

[JJ] Matrix of KC shown in Fig.1 using equation (1) is given by equation (4)

$$[JJ] = \begin{pmatrix}
0 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
2 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 3 & 3 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 3 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 3 \\
0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0
\end{pmatrix} \quad \text{----------------- (4)}$$

The characteristic polynomial coefficients for KC shown in Fig.1 (b)

The characteristic polynomial coefficients derived form [JJ] matrix of KC shown in Fig.1 (b) using MAT LAB software are:
0.0000 , 0.0000 , -0.0000, -0.0000 , 0.0000 , 0.0000 , -0.0001, -0.0004 , 0.0016 , 0.0108, -0.0059, -0.1238, -0.1160 , 0.5008, 0.8031 , -0.5288 , -1.1287

Structural Invariant for KC Shown In Fig.1 (b)

The set of structural invariant for KC Shown in Fig.1 (b), derived from [JJ] matrix using software MAT LAB are:
[∑JJ] = 3.2201e+010, [MJJ] =1.1287e+010
Our method reports that KC shown in Fig.1 (a) and Fig.1 (b) are non-isomorphic as the values of structural invariants [∑JJ] and [MJJ] are different. Note that by using another method Eigen vector [12] and artificial neural network [13], the same conclusion is obtained.

Illustrative Example -2 (Multidegree freedom chains)

The second example concerns two kinematic chains with 10 bars, 12 joints, three-degree of freedom as shown in Fig.2. The task is to examine whether these two chains are isomorphic.

[JJ] Matrix of KC shown in Fig.2 (a)

[JJ] Matrix of KC shown in Fig.2 (a) using equation (1) is given by equation (5).

$$[JJ] = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 2 & 0 & 0 & 0 \end{pmatrix} \text{ -------------------- (5)}$$

The characteristic polynomial coefficients for KC shown in Fig.2 (a)

The characteristic polynomial coefficients derived form [JJ] matrix of KC shown in Fig.1 (a) are:
0.0000, -0.0000, -0.0001, -0.0002, 0.0063, 0.0170, -0.1055, -0.3730, 0.4376, 2.1856, 0.0372, -3.5938, -1.6171

Structural Invariants for KC Shown In Fig.2 (a)

The set of structural invariant for KC Shown in Fig.2 (a), derived from [JJ] matrix are:
[∑JJ] = 8.3734e+006 [MJJ] =3.5938e+006

Similarly

[JJ] Matrix of KC shown in Fig.2 (b)

[JJ] Matrix of KC shown in Fig.2 (b) using equation (1) is given by equation (6).

$$[JJ] = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 \end{pmatrix} \quad \text{----------------} \ (6)$$

The characteristic polynomial coefficients for KC shown in Fig.2 (b)

The characteristic polynomial coefficients derived form [JJ] matrixes of KC shown in Fig.2 (b) are:
0.0000, 0 , -0.0001, -0.0002, 0.0058 , 0.0161, -0.0965, -0.3525, 0.4307, 2.2696, 0.1572, -2.9393, 0.7465

Structural Invariants for KC Shown In Fig.2 (b)

The set of structural invariant for KC Shown in Fig.2 (b), derived from [JJ] matrixes are:
[∑JJ] = 7.0147e+006 [MJJ] = 2.9393e+006
Our method reports that both the KC shown in Fig.2 (a) and Fig.2 (b) are non-isomorphic as the set of values of [∑JJ] and [MJJ] are different for both the KC. Note that by using other method summation polynomials [10], the same conclusion is obtained.

Illustrative Example -3

The third example concerns another two KC with 10 bars, 13 joints, single freedom as shown in Fig.3 (a) and Fig.3 (b).The task is to examine whether these two chains are isomorphic.

Structural Invariants for KC Shown In Fig.3 (a) and Fig.3 (b).

Following the same procedure, the structural invariants of the KC shown in Fig.3 (a) and Fig.3 (b) derived from the [JJ] matrices are written as:
For KC shown in Fig.3 (a): [∑JJ] = 7.5562e+007, [MJJ] = 3.5648e+007
For KC shown in Fig.3 (b): [∑JJ] = 7.5562e+007 , [MJJ] = 3.5648e+007
Our method reports that KC shown in Fig.3 (a) and Fig.3 (b) are isomorphic as the set of values of [∑JJ] and [MJJ] are same for both the KC. Note that by using another method artificial neural network [13], the same conclusion is obtained.

RESULTS

The proposed composite structural invariants [∑JJ] and [MJJ] of [JJ] matrix of the KC are able to detect isomorphism in the KC and even KC with co-spectral graphs. All the simple jointed 1-F, 8-links 16 KC and 1-F, 10-links 230 KC along with 2-F, 9-links 40 KC have been tested successfully for their non-isomorphism. The detailed results of composite structural invariants [∑JJ] and [MJJ] of [JJ] matrices of the above KC are with the authors.

CONCLUSION

In this paper, a simple, efficient, and reliable method to identify isomorphism is proposed. By this method, the isomorphism of mechanisms kinematic chains can easily be identified. It incorporates all features of the chain and as such, violation of the isomorphism test is rather difficult. In this method, the characteristic polynomials, composite structural invariants [∑JJ] and [MJJ] of [JJ] matrix of the KC are used. The advantage is that they are very easy to compute using MAT LAB software. It is not essential to determine both the composite invariants to compare the given KC , only in case the [∑JJ] is same then it is needed to determine [MJJ]for the KC. The [JJ] matrices can be written with very little effort, even by mere inspection of the KC. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar KC of one degree of freedom, but also KC of multi degree of freedom. The characteristic polynomials and composite structural invariants are very informative and from them valuable information regarding topology of kinematic chains can be predicted. The inner relation between characteristic value and mechanism KC need further study.

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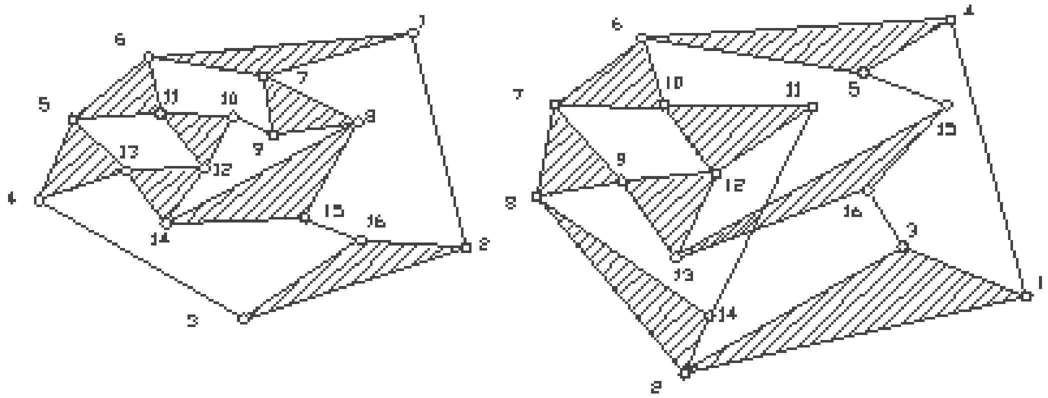
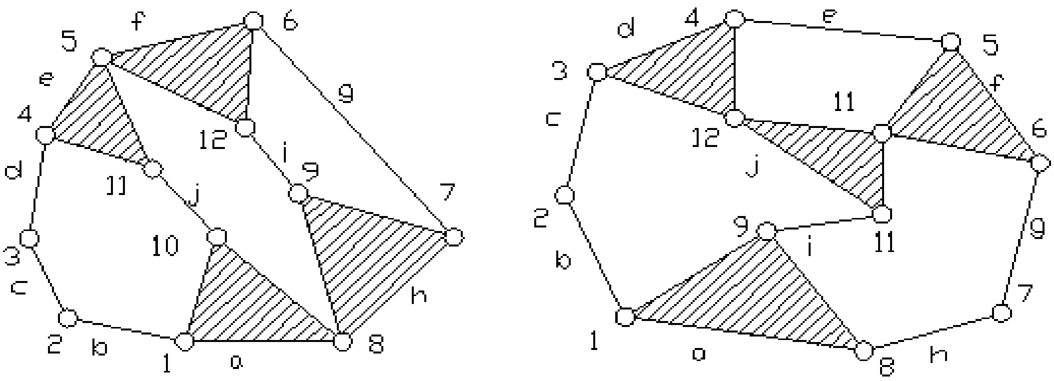


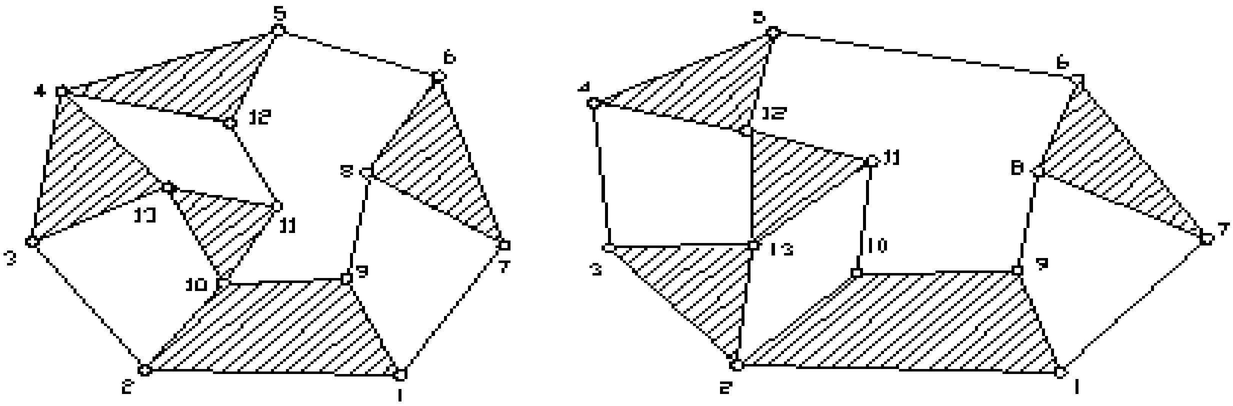
1(a) **1(b)**

Fig.1: Twelve-bar KC, single-degree of freedom.



2(a) **2(b)**

Fig.2: Ten-Bar KC, three-degree of freedom



3(a) **3(b)**

Fig.3: Ten-bar KC, single-degree of freedom