MATHEMATICAL MODELING APPLIED TO BURN INJURIES

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ABSTRACT
Mathematical Modeling is that branch of Mathematics, which has application in all the fields, ranging from chemical engineering to bio-medical sciences. Here, mathematical modeling has been applied in a biomedical problem. Heat injuries are a common problem of south Asia. Burn injuries are severe cases of heat injuries, which cause serious damage to the skin and body and in severe cases may result in death. We have tried to estimate the extent of burn injuries with the help of mathematical modeling. This study has a great importance as fluctuations in various parameters affecting the burn injuries can be mathematically analyzed. This would help in curing the burn injuries as the extent of burn injuries could be found mathematically.

Key words: Finite element method, Laplace transforms, Differential equations, Thermo regulation.

INTRODUCTION
Human body is endowed with a sophisticated mechanism of thermo- regulation. This phenomenon of thermo-regulation ensures that the core temperature of body is maintained at constant level despite wide ranging changes in the ambient temperature. The maintenance of body core temperature is important for its survival. The skin and subcutaneous (SST) region plays a significant role in this phenomenon.

When the skin surface is exposed to excessive direct heating for short duration or moderate heating for long duration the normal thermo regulatory process is disturbed, which causes burn injuries. Burns result in the most complex form of tissue injury and result in local lesions and physiological abnormalities. In these cases there might be an inordinate rise in body temperature resulting from the impairment and malfunctioning of the dermal system. The various physiological and physical factors involved can be studied effectively with the help of mathematical modeling of burn injuries. SST region plays a vital role in thermo heat regulation. This can be better understood by briefly explaining its physiology. The SST region is broadly divided into three sub layers, which are a) Epidermis b)Dermis c)Subcutaneous tissue region.

When burn injuries occur, there is a failure of defensive mechanism of thermoregulation and hence the tissue temperature rises above a threshold value for a finite interval of time[2]. The causative classification of burns can be made as :

1) Scalds
2) Flame burn
3) Electric burn
4) Radiation burn
5) Friction burn
6) Chemical burn

7) Lighting
8) Frost bite.

MATERIAL AND METHODS

Henrique and Moritz [1], derived the damage rate function, which tries to measure mathematically the damage caused by burns.
The damage rate function for burn injury at depth x from the skin surface is given by

$$\frac{d\Omega}{dt} = A\exp\left(\frac{\Delta E}{Ru(x,t)}\right) \qquad \text{(1)}$$

where A= frequency factor
E= activation energy
R= gas constant

The total injury at any point can be obtained by integrating the damage rate function over the duration of burn. Thus

$$\Omega(x) = A\int_0^t \exp\left[\frac{\Delta E}{Ru(x,t)}\right]dt$$

Ω identifies the injury thresholds. $\Omega = .53$ can cause irreversible epidermal damage where as at $\Omega = 10$ complete transepidermal necrosis would occur. The calculation of Ω would enable us in computation of damage due to burn injury.

An outline of human physiology of heat flow problems follows:
Skin and underlying subcutaneous tissues play an important role in the maintenance of the thermal conditions and core body temperature of humans and animals. While undergoing continuous thermal changes it regulate the heat through its system of various organs and subsystems. Thus, the study of human physiological heat flow is mainly the thermal study of the skin and sub dermal regions and the associated factors.

Body core, surface evaporation, and wind flow, atmospheric mechanisms, internal mechanism, are various factors affecting the heat flow in dermal regions.

The distribution of various constituents of the cutaneous and the adjoining regions affects the heat flow patterns. These constituents are epidermal layers, blood vessels, fat cells and muscles. The physical parameters such as thermal conductivity, specific heat and density of the medium depend on this distribution.

Mathematical Formulation and Solution:
If θ denotes the temperature of tissue at time t and at a distance x measured perpendicular into the tissue from the skin surface; then the rate of change of tissue temperature is given by

$$\rho \bar{c}\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial\theta}{\partial x}\right) + mc_b(\theta_a - \theta) + S \qquad \text{(1)}$$

Heat loss and gain at the skin surface due to convection, radiation and evaporation is given by

$$k\frac{\partial\theta}{\partial\eta} = h(\theta - \theta_0) + LE \qquad \text{(2)}$$

Where
ρ = tissue density
$\bar{c}$ = tissue specific heat

k = tissue thermal conductivity
m = mass flow rate of blood
c_b = Specific heat of blood
S = metabolic heat generation rate
η = Normal to outer boundary
h = heat transfer coefficient
θ_a = Climate temperature
L = latent heat evaporation
E = rate of sweat evaporation
θ_a = Arterial blood temperature

We would now look at the derivation of equation (1)

$$\rho \bar{c} \frac{\partial \theta}{\partial t}\bigg|_{conduction} = \frac{\partial}{\partial x}\left(k \frac{\partial \theta}{\partial x}\right) \text{...}(3)$$

$$\rho \bar{c} \frac{\partial \theta}{\partial t}\bigg|_{circulation} = mc_b(\theta_a - \theta) \text{...}(4)$$

$$\rho \bar{c} \frac{\partial \theta}{\partial t}\bigg|_{metabolism} = S \text{...}(5)$$

Proof of equation (3)

$P - Q = \delta x$

A = Area of cross section

Rate of change of heat in PQ = Heat inflow at P – Heat outflow at Q

$$= -Ak\frac{\partial \theta}{\partial t}\bigg|_x - \left(-Ak\frac{\partial \theta}{\partial x}\bigg|_{x+\delta x}\right) \text{..........................}(6)$$

As, heat flows from high temperature to low temperature so, when θ increases heat decreases so we have negative derivation. In equation (6)

$$-Ak\frac{\partial \theta}{\partial x}\bigg|_x$$ =mass(specific heat) temperature = mst

Quantity of heat in slab PQ = $(\rho A \delta x)c(\theta + 273)$ = (mass).(specificheat)(temperature)

Rate of change of heat in PQ = $\rho A \bar{c} \delta x \frac{\partial \theta}{\partial t}$

$$\rho A \bar{c} \delta \frac{\partial \theta}{\partial t} = -Ak\frac{\partial \theta}{\partial x}\bigg|_x - \left(-Ak\frac{\partial \theta}{\partial x}\bigg|\right)_{x+\delta x} = \frac{\partial}{\partial x}\left(Ak\frac{\partial \theta}{\partial x}\right)\delta x$$

In uniform rod.

$$\rho \bar{c} \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial \theta}{\partial x}\right)$$

Proof of equation (2)

Rate of change of heat due to blood flow
$= m\phi_A c_b \theta_A - m\phi_V c_b \theta_V = mc_b(\phi_A\theta_A - \phi_V\theta_V)$
m is the mass flow rate if $\phi_A = \phi_V$
where ϕ_A is the blood velocity in artery.
and ϕ_V is the blood velocity in vein.
So, Rate of change of heat due to blood flow = $mc_b\phi_A(\theta_A - \theta_V) = mc_b(\theta_A - \theta)$
Adding equation (3), (4) and (5)

$$\rho \bar{c}\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial \theta}{\partial x}\right) + mc_b(\theta_A - \theta) + S$$

Hence equation (1) is proved
Now we prove equation (2)
at the outer skin surface x = 0

$$k\frac{\partial \theta}{\partial \eta}\bigg|_{radiation} = H_r(\theta - \theta_a)......................(6)$$

$$k\frac{\partial \theta}{\partial \eta}\bigg|_{convection} = h_c(\theta - \theta_a)......................(7)$$

$$k\frac{\partial \theta}{\partial \eta}\bigg|_{evaporation} = LE................................(8)$$

Adding (6), (7), (8)
we get

$$k\frac{\partial \theta}{\partial \eta} = h(\theta - \theta_0) + LE \qquad \text{where } X\,\varepsilon[0, c]$$

The steady state case has been considered Saxena and Singh [3].
We consider the unsteady state case. We make use of finite element method.
<u>Steps in finite element method.</u>

(1) Construct a model.

$$\rho \bar{c}\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial \theta}{\partial x}\right) + mc_b(\theta_a - \theta) + S$$

$$k\frac{\partial \theta}{\partial \eta} = h(\theta - \theta_0) + LE$$

(2) Transformation to Euler –Lagrange variational form

$$\frac{\partial}{\partial x}\left(k\frac{\partial \theta}{\partial x}\right) + mc_b(\theta_a - \theta) + S - \rho \bar{c}\frac{\partial \theta}{\partial t} = 0$$

$$\frac{\partial F}{\partial \theta'} = k\theta' ....(a)$$

$$\frac{\partial F}{\partial \theta} = mc_b(\theta_a - \theta) + S - \rho \bar{c}\frac{\partial \theta}{\partial t} ....(b)$$

E – L variation form

$$\frac{\partial F}{\partial \theta}-\frac{d}{dx}\left(\frac{\partial F}{\partial \theta^{'}}\right)=0$$

Integrating (a) and (b)

$$F=\frac{k\theta^{'2}}{2}+\psi(\theta)\text{.............}(c)$$

$$F=\frac{-mc_b(\theta_A-\theta)^2}{2}+S\theta-\rho c\frac{\bar{\partial}\theta^2}{2\partial t}+\phi(\theta^{'})\text{............}(d)$$

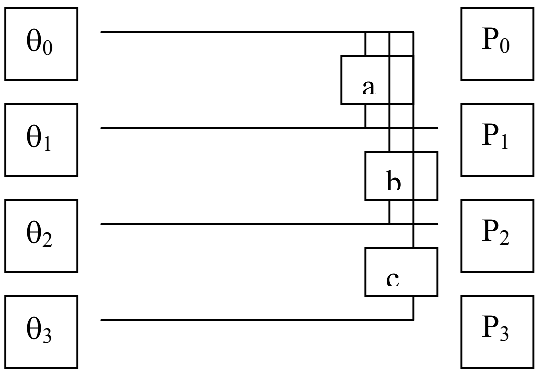
Comparing (c) and (d)

$$F=\frac{1}{2}\left[k\left(\frac{\partial\theta}{\partial x}\right)^2+mc_b(\theta_A-\theta)^2-2S\theta+\rho c\frac{\bar{\partial}\theta^2}{2\partial t}\right]$$

The partial equation coupled with equation (2) and compared with E-L equation we get the following form.

$$I=\frac{1}{2}\int_0^c\left[k\left(\frac{\partial\theta}{\partial x}\right)^2+mc_b(\theta_A-\theta)^2-2S\theta+\rho c\frac{\bar{\partial}\theta^2}{2\partial t}\right]dx+\frac{1}{2}h(\theta-\theta_a)^2+LE\theta$$

$$I=I_1+I_2+I_3=\sum_{i=1}^{3}I_i$$



(4) We consider the unsteady state case

$$I_1=\frac{1}{2}\int_0^a\left[k_1\left(\frac{\partial\theta^{(1)}}{\partial x}\right)^2+\rho\left(\frac{\partial\theta^{(1)^2}}{\partial t}\right)\right]dx+$$

$$\{\frac{1}{2}h(\theta^{(1)}-\theta_a)^2+LE\theta^{(1)}\}_{\theta^{(1)}=\theta_0}$$

$$I_2=\frac{1}{2}\int_a^b\left[k_2\left(\frac{\partial\theta^{(2)}}{\partial x}\right)^2+m_2c_b(\theta_A-\theta^{(2)})^2-2S_2\theta^{(2)}+\rho\frac{\partial\theta^{(2)^2}}{\partial t}\right]dx$$

$$I_3 = \int_b^c \left[k_3 \left(\frac{\partial \theta^{(3)}}{\partial x} \right)^2 + m_3 c_b \left(\theta_A - \theta^{(3)} \right)^2 - 2S_3 \theta^{(3)} + \rho \frac{\partial \theta^{(3)2}}{\partial t} \right] dx$$ (5) Assume

$\theta^{(i)} = A_i + B_i x$ i = 1, 2, 3

$k_1 = k$, $m_1 = 0$, $s_1 = 0$

(6) at x = 0 $\theta^{(1)} = A_1 = \theta_0$

at x = a $\theta^{(1)} = A_1 + B_1 a = \theta_1$

$\theta_0 = A_1$

$\theta_1 = A_1 + B_1 a = \theta_1$

$B_1 = \dfrac{\theta_1 - \theta_0}{a}$

$\theta^{(1)} = \theta_0 + \dfrac{\theta_1 - \theta_0}{a} x$

$\dfrac{\partial \theta^{(1)}}{\partial x} = \dfrac{\theta_1 - \theta_0}{a}$

at x = a $\theta_1 = A_2 + B_2 a$

at x = b $\theta_2 = A_2 + B_2 b$

In the dermis

$B_2 = \dfrac{\theta_2 - \theta_1}{b - a}$

$A_2 = \theta_1 - B_2 a = \dfrac{b\theta_1 - a\theta_2}{b - a}$

$\theta^{(2)} = \left(\dfrac{b\theta_1 - a\theta_2}{b - a} \right) + \left(\dfrac{\theta_2 - \theta_1}{b - a} \right) x$ $\dfrac{\partial \theta^{(2)}}{\partial x} = \dfrac{\theta_2 - \theta_1}{b - a}$

In the subdermis

$\theta_2 = A_3 + B_3 b$

$\theta_3 = A_3 + B_3 c$

$B_3 = \dfrac{\theta_3 - \theta_2}{c - b}$

$A_3 = \dfrac{c\theta_2 - b\theta_3}{c - b}$

$\theta^{(3)} = \dfrac{c\theta_2 - b\theta_3}{c - b} + \dfrac{\theta_3 - \theta_2}{c - b} x$

$\dfrac{\partial \theta^{(3)}}{\partial x} = \dfrac{\theta_3 - \theta_2}{c - b}$

So after doing several simplifications

$$I_1 = \frac{1}{2}\int_0^a \left[k\left(\frac{\theta_1 - \theta_0}{a}\right)^2 + \rho\bar{c}\frac{\partial}{\partial t}\left[\theta_0 + \frac{\theta_1 - \theta_0}{a}x\right]^2\right]dx + \frac{h}{2}(\theta_0 - \theta_a)^2 + LE\theta_0$$

$$I_1 = \frac{k}{2a}(\theta_1 - \theta_0)^2 + \frac{a\rho\bar{c}}{2}\frac{\partial}{\partial t}\left[\frac{(\theta_0 - \theta_1)^2}{3} + \theta_0{}^2 + \theta_0(\theta_1 - \theta_0)\right] + \frac{h}{2}(\theta_0 - \theta_a)^2 + LE\theta_0$$

or,

$$I_1 = \theta_0{}^2\left(\frac{k}{2a} + \frac{h}{2}\right) + \frac{k}{2a}\theta_1{}^2 - \frac{k}{a}\theta_0\theta_1 - h\theta_0\theta_a + \frac{h}{2}\theta_a{}^2 + LE\theta_0$$

Similarly

$$+\frac{a\rho\bar{c}}{6}\left[\frac{\partial\theta_1{}^2}{\partial t} + \frac{\partial\theta_0{}^2}{\partial t} + \frac{\partial\theta_1\theta_0}{\partial t}\right]$$

$$I_2 = \frac{1}{2}\int_a^b \left[\begin{array}{l} k_2\left[\frac{\theta_2 - \theta_1}{b-a}\right]^2 + m_2c_b\left(\theta_A - \left(\frac{b\theta_1 - a\theta_2}{b-a}\right) - \left(\frac{\theta_2 - \theta_1}{b-a}\right)x\right)^2 \\ -2S_2\left[\left(\frac{b\theta_1 - a\theta_2}{b-a}\right) + \left(\frac{\theta_2 - \theta_1}{b-a}\right)x\right] + \\ \rho\bar{c}\frac{\partial}{\partial t}\left[\left(\frac{b\theta_1 - a\theta_2}{b-a}\right) + \left(\frac{\theta_2 - \theta_1}{b-a}\right)x\right]^2 \end{array}\right]dx$$

or,

$$I_3 = \frac{1}{2}\int_b^c \left[\begin{array}{l} k_3\left[\frac{\theta_3 - \theta_2}{c-b}\right]^2 + m_3c_b\left(\theta_A - \left(\frac{c\theta_2 - b\theta_3}{c-b}\right) - \left(\frac{\theta_3 - \theta_2}{c-b}\right)x\right)^2 \\ -2S_2\left[\left(\frac{c\theta_2 - b\theta_3}{c-b}\right) + \left(\frac{\theta_3 - \theta_2}{c-b}\right)x\right] + \\ \rho\bar{c}\frac{\partial}{\partial t}\left[\left(\frac{c\theta_2 - b\theta_3}{c-b}\right) + \left(\frac{\theta_3 - \theta_2}{c-b}\right)x\right]^2 \end{array}\right]dx \quad I = I_1 + I_2 + I_3$$

$$\frac{\partial I}{\partial\theta_0} = 2\theta_0\left(\frac{k}{2a} + \frac{h}{2}\right) - h\theta_a + LE - \frac{k}{a}\theta_1 + \frac{\rho\bar{c}a}{3}\frac{\partial\theta_0}{\partial t} + \frac{\rho\overline{ca}}{6}\frac{\partial\theta_1}{\partial t}$$

$$\frac{\partial I}{\partial\theta_0} = 0$$

$$2\theta_0\left(\frac{k}{2a} + \frac{h}{2}\right) - h\theta_a + LE - \frac{k}{a}\theta_1 + \frac{\rho\bar{c}a}{3}\frac{\partial\theta_0}{\partial t} + \frac{\rho a\bar{c}}{6}\frac{\partial\theta_1}{\partial t} = 0 \text{..................(11)}$$

$$\frac{\partial I}{\partial\theta_1} = 0$$

$$\frac{\partial I}{\partial \theta_1} = 2\theta_1\left[\frac{k}{2a} + \frac{3k_2 + m_2 c_b\left(b^2 + a^2 - 2ab\right)}{6(b-a)}\right] - \frac{k}{a}\theta_0 + \theta_2\left[\frac{-6k_2 + b^2 + a^2 - 2ab}{6(b-a)}\right]$$

$$+\left[\frac{6ab\theta_A - 3\theta_A b^2 - 3\theta_A a^2 + 6S_2 ab - 3b^2 S_2 \ - 3a^2 S_2}{6(b-a)}\right] +$$

$$\rho\frac{\bar{c}}{6(b-a)}\left(2a^2 + 2b^2 - 2ab\right)\frac{\partial}{\partial \theta_1}\left(\theta_1 \frac{\partial \theta_1}{\partial t}\right) +$$ by calculus

$$\frac{a\rho\bar{c}}{6}\frac{\partial}{\partial \theta_1}\left(\frac{\partial \theta_1 \theta_0}{\partial t}\right) + \frac{\bar{c}\rho(b-a)}{6}\frac{\partial}{\partial \theta_1}\left(\frac{\partial \theta_1 \theta_2}{\partial t}\right) + \frac{a\rho\bar{c}}{3}\left(\frac{\partial \theta_1}{\partial t}\right)$$

$= 0$...(12)

of variations,

$$\frac{\partial}{\partial \theta_1}\left(\theta_1 \frac{\partial \theta_1}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial}{\partial \theta_1}\left(\theta_1^{\ 2}\right)\right) = 2\frac{\partial \theta_1}{\partial t}$$

$$\frac{\partial}{\partial \theta_1}\left(\frac{\partial \theta_1 \theta_0}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial}{\theta_1}\left(\theta_1 \theta_0\right)\right) = \frac{\partial \theta_0}{\partial t}$$

$$\frac{\partial}{\partial \theta_1}\left(\frac{\partial \theta_1 \theta_2}{\partial t}\right) = \frac{\partial \theta_2}{\partial t}$$

Equation (12) reduces to

$$\frac{\partial I}{\partial \theta_1} = 2\theta_1\left[\frac{k}{2a} + \frac{3k_2 + m_2 c_b\left(b^2 + a^2 - 2ab\right)}{6(b-a)}\right] - \frac{k}{a}\theta_0 + \theta_2\left[\frac{-6K_2 + b^2 + a^2 - 2ab}{6(b-a)}\right]$$

$$+\frac{6ab\theta_A - 3\theta_A b^2 - 3\theta_A a^2 + 6S_2 ab - 3b^2 S_2 - 3a^2 S_2}{6(b-a)}$$

$$+\frac{2\bar{c}\rho}{6(b-a)}\left(2a^2 + 2b^2 - 2ab\right)\frac{\partial \theta_1}{\partial t} + \frac{\rho\bar{c}}{6}\left(b-a\right)\frac{\partial \theta_2}{\partial t}$$

$$+a\frac{\bar{c}\rho}{3}\left[\frac{\partial \theta_1}{\partial t} + \frac{\partial \theta_0}{2\partial t}\right] = 0$$

$$\frac{\partial I}{\partial \theta_2} = 2\theta_2\left[\frac{3k_2 + m_2c_b\left(b^2 + a^2 - 2ab\right)}{6(b-a)} + \frac{3k_3 + m_3c_b\left(c^2 + b^2 - 2bc\right)}{6(c-b)}\right] +$$

$$\theta_1\left[\frac{-6k_2 + b^2 + a^2 - 2ab}{6(b-a)}\right]$$

$$+\theta_3\left[\frac{-6k_3 + c^2 + b^2 - 2bc}{6(c-b)}\right]$$

$$+(c-b)\left[\frac{6ab\theta_A - 3\theta_A a^2 - 3\theta_A b^2 + 6S_2ab - 3b^2S_2 - 3a^2S_2}{6(b-a)(c-b)}\right]$$

$$+(b-a)\left[\frac{6bc\theta_A - 3\theta_A b^2 - 3\theta_A c^2 + 6S_3bc - 3c^2S_3 - 3b^2S_3}{6(b-a)(c-b)}\right]$$

$$+\frac{\rho\bar{c}}{6(b-a)}\left[4\left(a^2 + b^2 - ab\right)\frac{\partial \theta_2}{\partial t} + (b-a)^2\frac{\partial \theta_1}{\partial t}\right]$$

$$+\frac{\rho\bar{c}}{6(c-b)}\left[4\left(c^2 + b^2 - bc\right)\frac{\partial \theta_2}{\partial t} + (c-b)^2\frac{\partial \theta_3}{\partial t}\right] = 0 \quad \text{...(13)}$$

Similarly,

$$\frac{\partial I}{\partial \theta_3} = 2\theta_3\left(\frac{3k_3 + m_3c_b\left(b^2 + c^2 - 2bc\right)}{6(b-c)}\right) + \theta_2\left(\frac{-6k_3 + c^2 + b^2 - 2bc}{6(c-b)}\right)$$

$$+\frac{6bc\theta_A - 3\theta_A b^2 - 3\theta_A c^2 + 6S_3bc - 3S_3b^2 - 3S_3c^2}{6(c-b)}$$

Applying Laplace

$$+\frac{\rho\bar{c}}{6(c-b)}\left[4\left(c^2 + b^2 - bc\right)\frac{\partial \theta_3}{\partial t} + (c-b)^2\frac{\partial \theta_2}{\partial t}\right] = 0$$

....(14)

transformation on both sides of equation (11), (12), (13), (14), and solving for L(θ_0), L(θ_1), L(θ_2), L(θ_3) and taking the inverse Laplace transforms the values of θ s are obtained.

RESULT AND DISCUSSION

Substituting,

k=.5*10**(-3), k2=1*10**(-3), k3=1*10**(-3), m2*cb=.525*10**(-3), m3*cb=.525*10**(-3), s2=.3*10**(-3),s3=.3*10**(-3), h=3*10**(-3), l=579, e=.16*10**(-3),le=579*.16*10**(-3), tb=37, a=.10, b=.36, c=.50

Then taking the inverse Laplace transforms we get,

$\theta_3 = 36.9954635$, $\theta_2 = 37.9258415$, $\theta_1 = 64.7401761$, $\theta_0 = 87.8700882$

The model helps in estimating the temperature from epidermis to sub-dermal regions of the skin. This estimate could be of immense value to plan the line of clinical treatment in burn-injury cases. Thus the modeling work would compliment and aid medical treatment. Also, since the estimate is deduced mathematically taking into cognizance of related and relevant variables (parameters) a fair estimate of temperature prevalent in the skin region is predicted and fed as a decision-facilitator to the medical as well as pharmaceutical personnel concerned for the treatment of burn injuries.

CONCLUSION

The temperature changes from the epidermis to the sub-dermal region from 87.87 degrees Celsius to 36.99 degree Celsius. So, with the help of mathematical modeling the extent of temperature changes and hence the extent of injuries incurred in the cells can be studied. This study has a great importance as fluctuations in various parameters affecting the burn injuries can be mathematically analyzed. This would help in curing the burn injuries as the extent of burn injuries could be found mathematically.

REFERENCES

1. Henriques, F. C. and Moritz, A. R., Studies of thermal injury, 1947. The conduction of heat to and through skin and the temperature attained therein. A theoretical and an experimental investigation. Amer. J. Pathol., 23, 531-549.

2. Saxena, V. P., Temperature distribution in human skin and sub-dermal tissues, 1983. J. Theo. Biol., 102, 277-286.

3. Saxena V. P., Yadav Anoop Singh Temperature distribution in human skin and subcutaneous tissue with burn injury due to hot climate-steady state case, 1998. Bio-Science Research Bulletin, Vol. 14: 2, 51-58.